MAT 119 Fall 09 Instructor: M. Keindl **Linear Programming Worksheet** **Names:**

1. A Division of Winston Furniture Company manufactures dining tables and chairs. Each table requires 40 board feet of wood and 3 labor-hours. Each chair requires 16 board feet of wood and 4 labor-hours. The profit for each table is \$45, and the profit for each chair is \$20. In a certain week, the company has 3200 board feet of wood available and 520 labor-hours available. How many tables and chairs should Winston manufacture in order to maximize its profit? What is the maximum profit?

	board feet of wood	labor hours
table	40	3
chair	16	4
	$\leq$ 3200	$\leq$ 520

Variables:

x : # of tables

y : # of chairs

Objective function: $z = 45x + 20y$

Constraints:

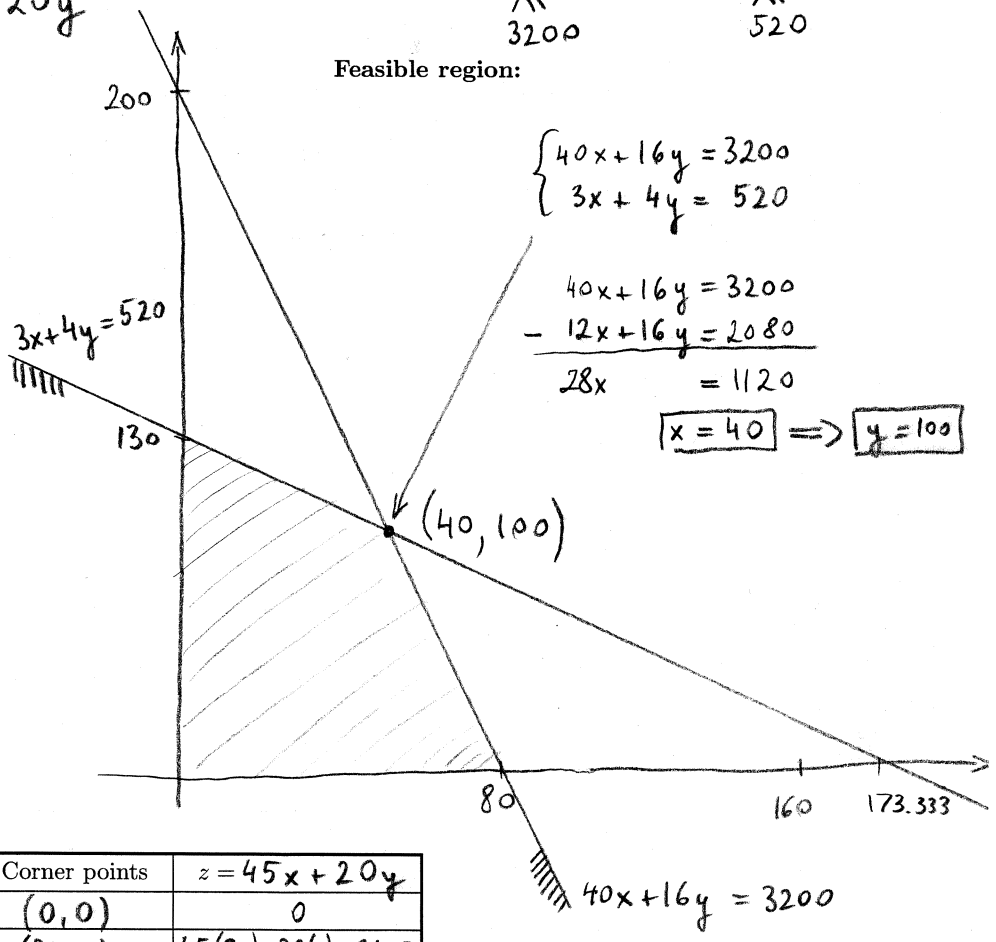
$$40x + 16y \leqslant 3200$$

$$3x + 4y \leqslant 520$$

$$x \geqslant 0$$

$$y \geqslant 0$$

Feasible region:



$$\begin{cases} 40x + 16y = 3200 \\ 3x + 4y = 520 \end{cases}$$

$$40x + 16y = 3200$$
$$-\ 12x + 16y = 2080$$
$$28x = 1120$$

$x = 40 \Rightarrow y = 100$

Corner points	$z = 45x + 20y$
(0, 0)	0
(80, 0)	$45(80)+20(0) = 3600$
(40, 100)	$45(40)+20(100) = 3800$ ← max
(0, 130)	$45(0)+20(130) = 2600$

Conclusion: Winston should manufacture 40 tables and 100 chairs for a maximum profit of \$3800.

2. An investment broker wants to invest up to \$20,000. She can purchase a type A bond yielding a 10% return on the amount invested, and she can purchase a type B bond yielding a 15% return on the amount invested. She wants to invest at least as much in the type A bond as in the type B bond. She will also invest at least \$5,000 in the type A bond and no more than \$8,000 in the type B bond. How much should she invest in each type of bond to maximize her return?

Variables:

x: amount invested in type A

y: amount invested in type B

Objective function: $z = .1x + .15y$

Constraints:

$x \geq y$

$x \geq 5{,}000$

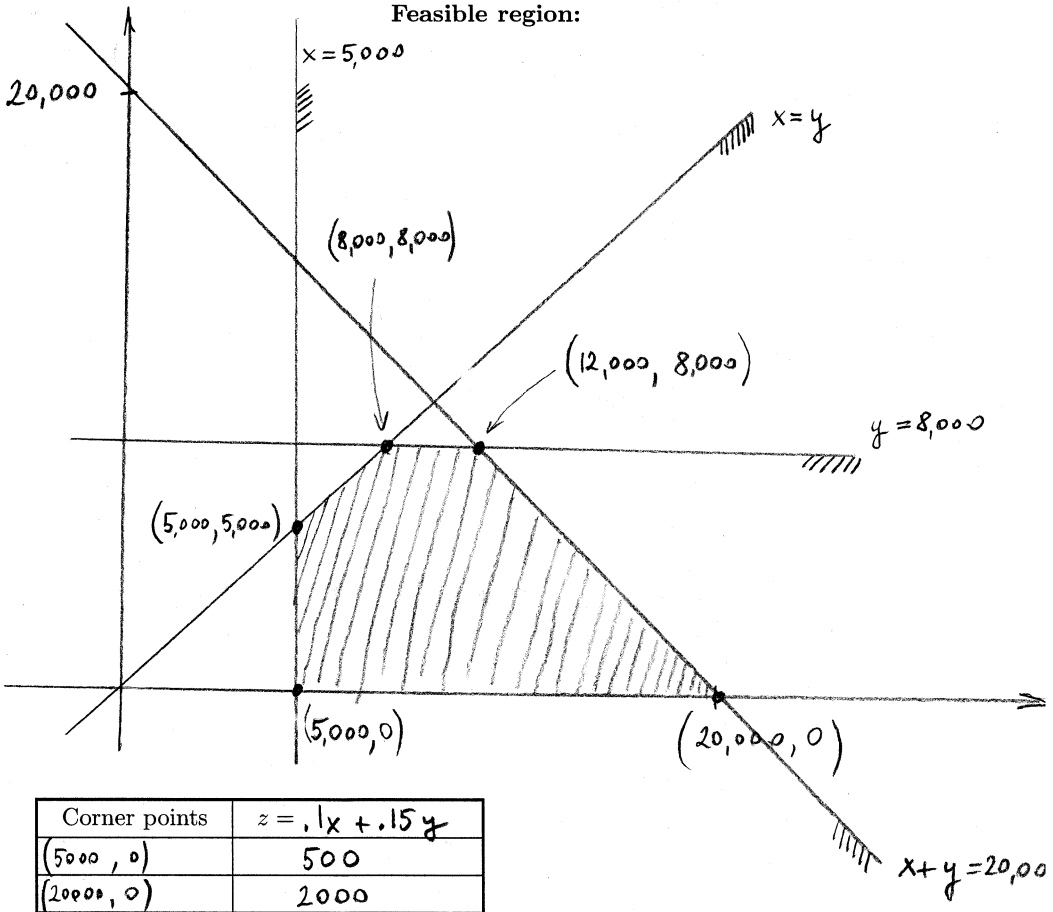
$y \leq 8{,}000$

$x + y \leq 20{,}000$

$x \geq 0$

$y \geq 0$

Feasible region:



Corner points	$z = .1x + .15y$	
(5000, 0)	500	
(20000, 0)	2000	
(12000, 8000)	2400	← max
(8000, 8000)	2000	
(5000, 5000)	1250	

Conclusion: She should invest \$12,000 in type A bond and \$8,000 in type B bond for a maximum return of \$2400.

3. A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain a minimum of 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 4 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron, and vitamin C are met.

Variables:

x: # of ounces from food A

y: # of ounces from food B

	calcium	iron	Vit C	chol.
food A	30 mg	1 mg	2 mg	2 mg
food B	25 mg	.5 mg	5 mg	4 mg
	$\geq$ 400 mg	$\geq$ 10 mg	$\geq$ 40 mg	

Objective function: $z = 2x + 4y$

Constraints:

$$30x + 25y \geq 400$$

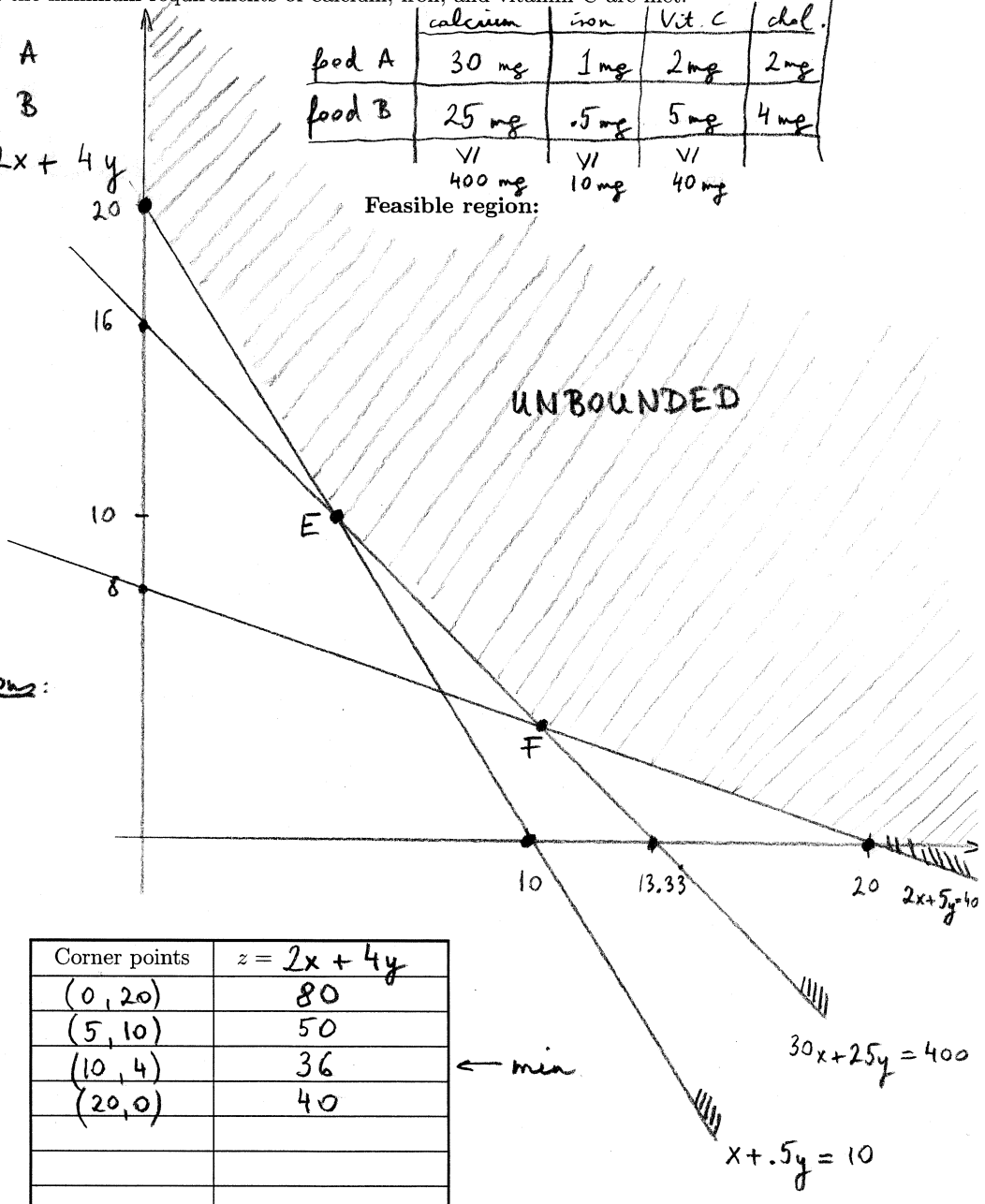
$$x + .5y \geq 10$$

$$2x + 5y \geq 40$$

$$x \geq 0$$

$$y \geq 0$$

Feasible region:



Corner point calculations:

E: $x + .5y = 10$, $30x + 25y = 400$

$30x + 15y = 300$

$30x + 25y = 400$

$10y = 100$

$y = 10 \Rightarrow x = 5$

F: $30x + 25y = 400$, $2x + 5y = 40$

$30x + 25y = 400$

$10x + 25y = 200$

$20x = 200$

$x = 10$, $y = 4$

Corner points	$z = 2x + 4y$	
(0, 20)	80	
(5, 10)	50	
(10, 4)	36	← min
(20, 0)	40	

Conclusion:

10 ounces of food A and 4 ounces of food B should be prepared in order to minimize cholesterol content but still satisfy nutrition requirements.

4. Bata Aerobics manufactures two models of steppers used for aerobic exercises. Manufacturing each luxury model requires 10 lb of plastic and 10 min of labor. Manufacturing each standard model requires 16 lb of plastic and 8 min of labor. The profit for each luxury model is \$40, and the profit for each standard model is \$30. If 6000 lb of plastic and 60 labor-hours are available for the production of the steppers per day, how many steppers of each model should Bata produce each day in order to maximize its profit?

	plastic	labor
luxury	10 lb	10 min
standard	16 lb	8 min
	≤ 6000	≤ 60 hours = 3600 min

Variables:

x: # of luxury steppers

y: # of standard steppers

Objective function: $z = 40x + 30y$

Constraints:

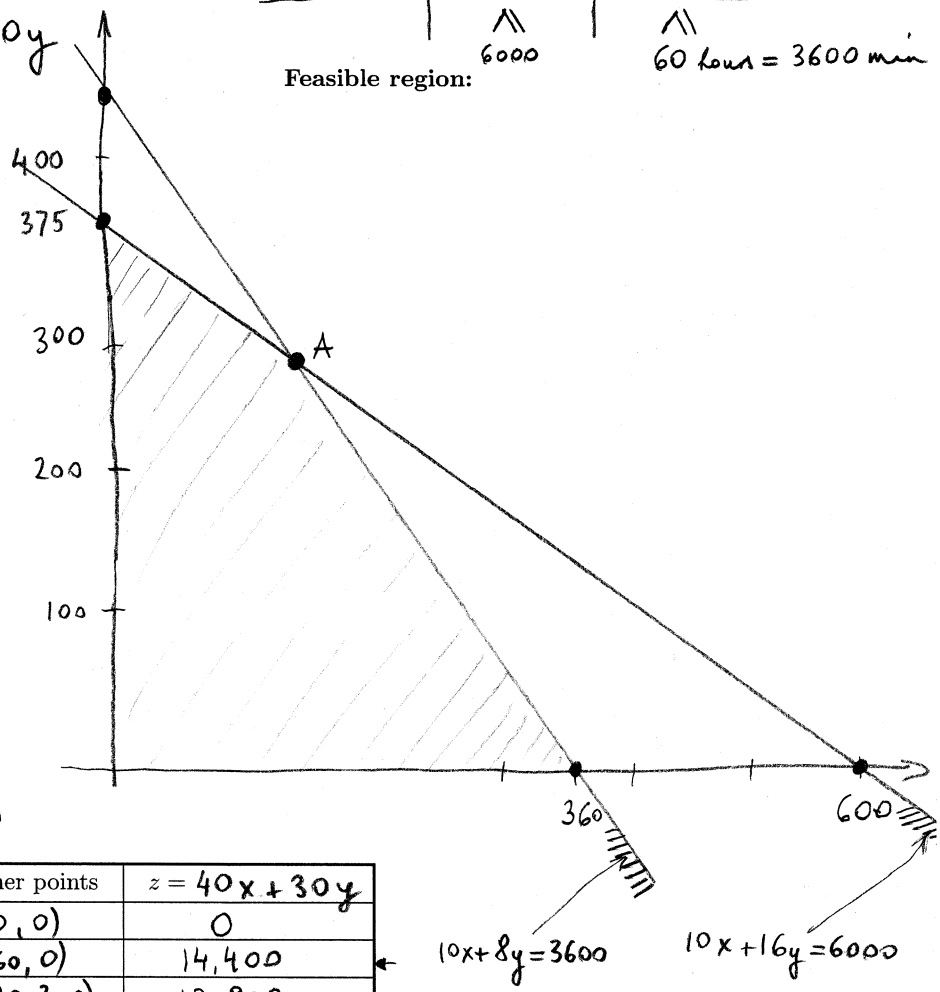
$$10x + 16y \leq 6000$$

$$10x + 8y \leq 3600$$

$$x \geq 0$$

$$y \geq 0$$

Feasible region:



Corner point calculation:

A $\begin{cases} 10x + 16y = 6000 \\ 10x + 8y = 3600 \end{cases}$

$8y = 2400$

$y = 300 \Rightarrow x = 120$

Corner points	$z = 40x + 30y$
(0, 0)	0
(360, 0)	14,400
(120, 300)	13,800
(0, 375)	11,250

Conclusion:

Bata should produce 360 luxury steppers per day in order to maximize its profits.