In[17]:=
```
(* We find the maximum of f(x,y) = x y on the circle x^2+y^2 = 1 *)
f = x y
g = x^2 + y^2
ContourPlot[f, {x, -1, 1}, {y, -1, 1}]
ContourPlot[{g == 1}, {x, -1, 1}, {y, -1, 1}]
```

Out[17]= $x\,y$

Out[18]= $x^2 + y^2$

In[23]:=
```
Gf = Grad[f, {x, y}]
Gg = Grad[g, {x, y}]
```

Out[23]= $\{y, x\}$

Out[24]= $\{2\,x, 2\,y\}$

In[30]:=
```
sol = Solve[{Gf == alpha Gg, g == 1}]
```

Out[30]= $\left\{\left\{\text{alpha} \to -\frac{1}{2}, x \to -\frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}, \left\{\text{alpha} \to -\frac{1}{2}, x \to \frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}, \left\{\text{alpha} \to \frac{1}{2}, x \to -\frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}, \left\{\text{alpha} \to \frac{1}{2}, x \to \frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}\right\}$

In[26]:=
```
s1 = {x, y, f} /. sol[[1]]
s1 // N
```

Out[26]= $\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{2}\right\}$

Out[27]= $\{-0.707107, 0.707107, -0.5\}$

In[28]:=
```
s2 = {x, y, f} /. sol[[2]]
s2 // N
```

Out[28]= $\left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{2}\right\}$

In[35]:=
```
s3 = {x, y, f} /. sol[[3]]
s3 // N
```

Out[35]= $\left\{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{2}\right\}$

Out[36]= $\{-0.707107, -0.707107, 0.5\}$

In[39]:=
```
s4 = {x, y, f} /. sol[[4]]
s4 // N
```

Out[39]= $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right\}$

Out[40]= $\{0.707107, 0.707107, 0.5\}$

```
(* Global maximum of 0.5 at (1/sqrt(2),1/sqrt(2)) and (1/sqrt(2),1/sqrt(2)) *)
(* Global minimum of -
 0.5 at (1/sqrt(2),-1/sqrt(2)) and (-1/sqrt(2),1/sqrt(2)) *)
```