```
(* We compute the line integral of F on
 the upper semicircle with radius 2 in two ways. *)
```

In[8]:=
```
(* direct calculation *)
R = 2
F = {y, 2 x - y}
(* parametrization of the semicircle *)
r = {R Cos[t], R Sin[t]}
(* F(r(t)) *)
pF = F /. {x → r[[1]], y → r[[2]]}
D[r, t]
(* F(r(t)).r'(t) *)
g = Dot[pF, D[r, t]]
(* line integral *)
li1 = Integrate[g, {t, 0, Pi}]
```

Out[8]= 2

Out[9]= {y, 2 x - y}

Out[10]= {2 Cos[t], 2 Sin[t]}

Out[11]= {2 Sin[t], 4 Cos[t] - 2 Sin[t]}

Out[12]= {-2 Sin[t], 2 Cos[t]}

Out[13]= 2 Cos[t] (4 Cos[t] - 2 Sin[t]) - 4 Sin[t]$^2$

Out[14]= 2 $\pi$

In[37]:=
```
(* Using Green's Theorem *)
(* parametrization of the line on the bottom *)
s = {t, 0}
(* F(s(t)) *)
pF = F /. {x → s[[1]], y → s[[2]]}
D[s, t]
(* F(s(t)).s'(t) *)
g = Dot[pF, D[s, t]]
(* line integral *)
li2 = Integrate[g, {t, -R, R}]
Curl[F, {x, y}]
(* double integral of curl on the interior *)
di = R^2 Pi / 2 Curl[F, {x, y}]
(* line integral on r = double integral of curl - line integral on s *)
di - li2
```

Out[37]= $\{t, 0\}$

Out[38]= $\{0, 2\,t\}$

Out[39]= $\{1, 0\}$

Out[40]= $0$

Out[41]= $0$

Out[42]= $1$

Out[43]= $2\,\pi$

Out[44]= $2\,\pi$