1. Show that in $\triangle ABC$ we have

$$\frac{3s}{2} \leq m_a + m_b + m_c \leq 2s.$$

Hint: use the $\triangle$ inequality.

Solution: Use the notation of the figure.

The $\triangle$ inequality for $\triangle GM_bM_c$ gives

$$\frac{1}{3}m_b + \frac{1}{3}m_c \geq \frac{a}{2}.$$ 

Similarly we can get

$$\frac{1}{3}m_a + \frac{1}{3}m_c \geq \frac{b}{2},$$

$$\frac{1}{3}m_a + \frac{1}{3}m_a \geq \frac{c}{2}.$$ 

Adding the inequalities gives

$$m_a + m_b + m_c \geq \frac{3s}{2}.$$ 

The $\triangle$ inequality for $\triangle M_bM_cV$ gives

$$m_c \leq \frac{a}{2} + \frac{b}{2}.$$ 

Similarly we can get

$$m_a \leq \frac{b}{2} + \frac{c}{2},$$

$$m_b \leq \frac{a}{2} + \frac{c}{2}.$$ 

Adding the inequalities gives

$$m_a + m_b + m_c \leq 2s.$$