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Problem 1. Show that $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in \mathbb{R}$.

Solution. For all $a, b \in \mathbb{R}$ we have

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + ab + ba + b^2 \quad (\text{since } ab = ba)\\ &= a^2 + 2ab + b^2.\end{aligned}$$

□

Problem 2. Find two numbers with a sum of 3 and a product of 2.

Solution. Denote the two numbers by a and b. Since the sum of the numbers is 3, we must have

$$a + b = 3.$$

Since their product is 2, we must have

$$ab = 2.$$

From the first equation we have

$$a = 3 - b.$$

Substituting this into the second equation gives

$$(3-b)b = 2.$$

This implies

$$(b-1)(b-2) = b^2 - 3b + 2 = 0.$$

Hence $b \in \{1, 2\}$. If $b = 1$ then $a = 2$. If $b = 2$ then $a = 1$. So the only possibility is that one of the numbers is 1 while the other is 2. These two numbers in fact have a sum of 3 and a product of 2. □

Problem 3. (GAP) Find the sum of the primes between 10 and 10000.

Solution. The following GAP code was used. We use a loop to test the integers between 10 and 10000. Whenever we find a prime, we add it to a partial sum variable *sum* that is initialized to 0 at line 1.

```
1 sum:=0;; # partial sum
2 for i in [10..10000] do
3   if IsPrime(i) then
4     sum := sum+i;
5   fi;
6 od;
7 sum;
```

This produced the following output:

```
5736379
```

So the sum of the primes between 10 and 10000 is 5736379. □

Problem 4. (GAP) Consider the subgroup $H := \langle (123), (1,2) \rangle$ of S_4. Find H and the coset $(14)H$.

Solution. The following GAP code was used.

```
1 G:=SymmetricGroup(4);;
2 H:=Subgroup(G,[(1,2,3)(4)]);;
3 StructureDescription(H);
4 Elements(H);
5 Elements(H)*(1,4);
```

This produced the following output:

```
"S3"
[ (), (2,3), (1,2), (1,2,3), (1,3,2), (1,3) ]
[ (1,4), (1,4)(2,3), (1,2,4), (1,2,3,4), (1,3,2,4), (1,3,4) ]
```

So

$$H \cong S_3 = \{(), (23), (12), (123), (132), (13)\}$$

and

$$(14)H = \{(14), (14)(23), (124), (1234), (1324), (134)\}.$$

Note that GAP multiplies permutations from left to right, so GAP's right cosets are our left cosets. □