Project 1 - 362

Due January 30th, 2003

Explore the uses of Bisection, Fixed Point Iteration, and Newton's method algorithms for solving problems. The following list contains suggestions as to the components of your project (observe guidelines).

1. Pick a simple problem such as $f(x) = x^2 - 1 = 0$ and apply all bisection and fixed point (Newton) methods to obtain approximate solutions.
2. Compare algorithm results to each other and the known correct answer.
3. Discuss accuracy, convergence, ease of coding.
4. Test each algorithm on problems which "fail", such as for example Newton's method for $f(x) = x^2 = 0$.
5. Consider problems with multiple answers, such as $f(x) = (x + 1)x(x - 1)(x - 2) = 0$. Which initial guesses go to which answer? Which algorithm works best if you want all answers?
6. For a given root problem $f(x) = 0$, try various functions g such that a fixed point satisfying $g(x) = x$. Which choices lead to convergence? Which converge faster? Observe the relationship of the bound k on $|g'(x)|$ over the interval in question to the speed of convergence.
7. Remember to try something creative, extra, fun, unexpected! Try a problem with unknown answer and convince me that you have a good approximation.

Challenge: Let $f(z) = z^3 + 1$ be a function from $\mathbb{C}$ to $\mathbb{C}$ (Complex plane). Using complex multiplication and division, implement Newton's method to find the 3 complex zeroes. A fascinating diagram can be made by doing the following: For each complex number in the square $\{a + bi : -1 \leq a, b \leq 1\}$ plot a point in one of 3 colors, depending on which of the 3 roots that initial guess leads too. (Basins of attraction with Fractal boundary)