Q3 MAT 362

Work 4 problems, omitting any 1; clearly mark omitted problem.

1. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = x^3$ and $\mathbf{x} = \{0, 1, 2\}$. Compute the Lagrange Interpolating Polynomial fitting f at the $\{x_0, x_1, x_2\}$.

2. Using the above polynomial, approximate $f(\frac{1}{2})$ and compare the actual error to the error estimate obtained by using the Lagrange Interpolating Polynomial error term.

3. Estimate $f'(2)$ using a 3-point backwards 1^{st} difference scheme; compare the actual error to the error estimate obtained by using the scheme's error term.

4. Pseudocode carefully a tridiagonal system solver. Define all terms and use proper notation.

5. Solve $Ax = b$ for x where $b = (1, 1, 1)^T$ and A is the $2^{\mathbf{nd}}$ 3×3 difference matrix for zero boundry conditions on the interval $[0, 1]$ given by

$$A = \begin{pmatrix} 32 & -16 & 0 \\ -16 & 32 & -16 \\ 0 & -16 & 32 \end{pmatrix}.$$