NORTHERN ARIZONA UNIVERSITY

Department of Mathematics and Statistics

COLLOQUIUM - Tuesday, October 14th, 2014

4:00 – 5:00 pm, Adel Mathematics Bldg., Room 164 (refreshments at 3:45)

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Holomorphic Functions in Volatility Models in Mathematical Finance

Abstract

We begin by a brief discussion about the role of ***rigorous*** mathematical models in a ***stochastic*** environment of finance; this attempt being termed ***"Mathematical Finance"***. We present a simple model for determining the price of the European option on a stock (having certain price) in a stochastic environment represented mainly by *stochastic volatility*. The two unknown functions in this model are the *option price* and the *stochastic volatility* ($= \sqrt{\text{variance}}$); of course, the two are correlated. Using the so-called ***no arbitrage*** pricing approach (guaranteeing a "fair price" which excludes risk-free profits) pioneered in the papers by BLACK and SCHOLES (1972, 1973) and MERTON (1973), we obtain a simple parabolic partial differential equation for the option price, p, as a function depending on two space variables, $x = \log S$ – the *logarithmic stock price* and v – the *variance*, and on the time variable, t, i.e., $p = p(x, v, t)$. The variance v ($=$ volatility^2) at time t can be determined from the statistical data of the market, so the remaining problem is to study the function $x \mapsto p(x, v, t) : \mathbb{R}^1 \to \mathbb{R}$ and its inverse function. Fortunately, one can prove that p can be extended to a *holomorphic function* (in a complex domain $\Omega \subset \mathbb{C}^3$) of all its variables. In the final part of this lecture we discuss the applications of the holomorphic extension to the problem of *complete markets* in Mathematical Finance, i.e., can one determine (uniquely) the stock price from the option price ?

Algebra Combinatorics Geometry and Topology (ACGT) Seminar: Tuesday October 14th, 12:45 – 1:45 pm, AMB 164.

Applied Math Seminar (AMS): Thursday, October 16th, 12:45 – 1:45 pm, AMB 164.

Friday Afternoon Undergraduate Mathematics Seminar (FAMUS) meets Friday at 3pm.