Test 2 316

Work 2 of the 4, indicating which 2 are to be graded. *No notes, books, or calculators.*

(1) Let P_2 denote the set of all polynomials of degree less than or equal to 2. Determine if the following sets H are a subspace of P_2, and if so, determine a basis for H and the dimension of H.
(a) $H = \{p \in P_2 | p(-t) = p(t)\}$

(b) $H = \{p \in P_2 | p'(-1) = 0\}$

(c) $H = \{p \in P_2 | \int_0^1 tp(t)\,dt = 0\}$

(2) Consider the parallelogram P determined by $u = (1, 2)^T$ and $v = (-2, 1)^T$.
(a) Determine the area of P.

(b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ has standard matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, compute the area of the parallelogram $T(P)$ *using the determinant of* A.

(3) Let $T : P_1 \to P_1$ be defined by $T(a + bx) = ax + b$.

(a) Find a basis for $\ker(T)$.

(b) What is the standard basis for P_1? Write down the corresponding standard matrix for T.

(c) If possible, compute $T^{-1}(a + bx)$.

(4) Let V be a finite dimensional vector space with two subspaces X and Y.

(a) Show that $X \cap Y$ is also a subspace of V.

(b) Give an example of V, X, and Y showing that a basis for $X \cap Y$ need not contain elements of the bases for X or Y.