Q3 316

Work 3 of 4 parts of 2 of 3 problems; mark through omitted parts and problems.
No notes, books, or calculators.

(1) For the given $m \times n$ matrices A, consider the associated linear transformations $T : \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(x) = Ax$. Determine if T is 1-1, onto, neither 1-1 nor onto, or both 1-1 and onto. *If* T is not 1-1, find two vectors in $\mathbb{R}^n$ that map to the same output in $\mathbb{R}^m$ (may I suggest $0 \in \mathbb{R}^m$). *If* T is not onto, find a vector that is not in the range. (*Omit 1 of a-d*).

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(2) For each of the Linear Transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$, find the associated standard 2×2 matrix A (*omit 1 of a-d*):

(a) T rotates vectors $45°$ counter-clockwise.

(b) T stretches $\hat{i}$ by 1 and $\hat{j}$ by 2.

(c) T projects vectors onto the line $y = x$.

(d) T reflects vectors about the line $y = x$.

(3) Provide careful definitions of the following (*omit 1 of a-d*):

(a) What it means for a function $T : \mathbb{R}^n \to \mathbb{R}^m$ to be a linear transformation.

(b) What it means for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ to be 1-1.

(c) What it means for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ to be onto.

(d) What it means for a collection of vectors $\{u_1, \ldots, u_n\}$ in $\mathbb{R}^m$ to be linearly independent.