Q4 316

Work Problem 1 or 2; mark through omitted problem.

No notes, books, or calculators.

(1) If possible, find A^{-1}. Verify your answer by computing AA^{-1}.

(a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

(2) Let $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and consider a linear transformation

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

where $T(u) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T(v) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) Write down the standard matrix A^{-1} for T^{-1} and use to compute the standard matrix A for T.

(b) Find $c_1, c_2 \in \mathbb{R}$ so that $\hat{i} = c_1 u + c_2 v$ and $d_1, d_2 \in \mathbb{R}$ so that $\hat{j} = d_1 u + d_2 v$.

(c) Compute $T(\hat{i})$ and $T(\hat{j})$.

(d) Write down the standard matrix A for T and compute Au and Av.