HW 2 - MAT661

due: Wed 2/15/06.

Consider the following functions $f : \mathbb{R} \to \mathbb{R}$:

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 2$, $p_0 = 1$.

(a) Perform 3 iterations by hand of Newton's method to approximate a zero of f.

(b) Does this f satisfy the hypothesis of our Newton convergence theorem (Problem 0.12)?

(c) Code Newton's method and output the first 10 iterations; compare your approximations to an exact zero p of f. Output $\frac{|p_{k+1}-p|}{|p_k-p|^\alpha}$ for $\alpha = 0, 1, 2, 3$.

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$, $p_0 = 1$.

(a) Perform 3 iterations by hand of Newton's method to approximate a zero of f.

(b) Does this f satisfy the hypothesis of our Newton convergence theorem (Problem 0.12)?

(c) Code Newton's method and output the first 10 iterations; compare your approximations to an exact zero p of f. Output $\frac{|p_{k+1}-p|}{|p_k-p|^\alpha}$ for $\alpha = 0, 1, 2, 3$.

Consider the following $f : \mathbb{R}^2 \to \mathbb{R}^2$. Here, Newton's method is $z^{k+1} = z^k - (J(z^k))^{-1} f(z^k)$, where $z = \begin{bmatrix} x \\ y \end{bmatrix}$, $f = \nabla h$ for the given function $h : \mathbb{R}^2 \to \mathbb{R}$, and J is the Jacobian of f (also known as the Hessian of h) given by $J = \begin{bmatrix} \frac{\partial^2 h}{\partial x^2} & \frac{\partial^2 h}{\partial x \partial y} \\ \frac{\partial^2 h}{\partial y \partial x} & \frac{\partial^2 h}{\partial y^2} \end{bmatrix}$.

(1) Let $h(x, y) = (x^2 + y^2)(1 - (x^2 + y^2))$.
  (a) Graph h and find analytically the critical points of h.

  (b) Write a program to implement Newton's method to find some of these critical points.

  (c) Discuss the convergence. Analyze the critical points (minima, maxima, saddle, degenerate).

(2) Let $h(x, y) = (x^2 + 2y^2)(1 - (x^2 + y^2))$.
  (a) Graph h and find analytically the critical points of h.

  (b) Write a program to implement Newton's method to find some (all?) of these critical points.

  (c) Discuss the convergence. Analyze the critical points (minima, maxima, saddle, degenerate).