"MINI PROJECT" 1 - MAT 661

due: Wednesday 1/25/06.

Find the first four smallest eigenvalues and corresponding eigenvectors of the $(n-1) \times (n-1)$ tridiagonal matrix

$$n^2 \begin{bmatrix} 2 & -1 & 0 & \cdots & & 0 \\ -1 & 2 & -1 & & & 0 \\ 0 & -1 & 2 & & & 0 \\ \vdots & & & & & \vdots \\ 0 & & & & 2 & -1 \\ 0 & \cdots & & & -1 & 2 \end{bmatrix}.$$

Use a matrix with $n = 10, 100$, and 1000 defining the number (one less) of rows and columns. For each eigenvector $y = (y_1, \ldots, y_{n-1})$ plot the points (i, y_i) for $i = 1, \ldots, n-1$. You may add $y_0 = 0$ and $y_n = 0$ to make vectors of size $n+1$, for better pictures.

Hint: The plots and the eigenvalues will look familiar. This tridiagonal matrix "approximates the negative second derivative" of the discretization of a function. Can you predict the exact eigenvalues and eigenfunctions / eigenvectors? What about the 4 largest eigenvalues and corresponding eigenvectors?