"MINI PROJECT" 4 - MAT 661

due: Friday 3/31/06.

Consider the Heat Equation

$$u_t = u_{xx}, \ t > 0, \ x \in (0,1)$$

with zero Dirichlet boundary conditions

$$u(t,0) = 0 = u(t,1)$$

and initial temperature distribution

$$u(0,x) = f(x).$$

(1) Code and *test* Explicit solver, for $f(x) = \sin \pi x$. What relationship between Δx and Δt is required to observe relevant approximations?

(2) Repeat using the Implicit solver, demonstrating that the method is unconditionally stable.

(3) Repeat using $f(x) = \sin \pi x + .1 \sin 10\pi x$.

(4) Repeat using the piecewise defined "tent function" from your Fourier series homework for f.

You have lots of options for presenting your results! Time slices are good, but you can also give a 3-D plot of u versus $x - t$. Can you compare your solution to the actual solution?

Some ideas for further considerations: zero Neuman (insulated) boundary conditions, convection term cu_x added to right hand side, two space dimensions ($(x,y) \in (0,1) \times (0,1)$ with $u_t = u_{xx} + u_{yy}$), and so on.