Q4 661

Work 3 of the 6 problems, clearly marking which problems are to be graded. No notes, books, or calculators.

(1) Consider the 'transport' PDE $u_t + cu_x = 0$, $x \in \mathbb{R}$, $t > 0$, for a given constant $c > 0$, with . Write down and verify a solution to the corresponding IVP given by $u(0, x) = f(x)$.

(2) Modify the linear PDE from Problem (1) to be a nonlinear PDE for $x \in (0, 1)$ with boundary conditions $u_x(t, 0) = 0 = u_x(t, 1)$ which can produce shocks. Give an example of an initial 'density' f that can lead to shocks. Pseudocode a numerical solver and briefly discuss numerical difficulties that are likely to arise.

(3) State carefully the Heat Equation in time and one space variable $x \in (0, 1)$ with zero boundary conditions and a specified initial temperature distribution f.

(4) Consider dividing the space interval $(0, 1)$ into $n = 4$ intervals, and take the initial temperature distribution to be given by $f(x) = \sin \pi x$.

(a) Perform one step of the Explicit method to approximate the temperature u at time $t = \Delta t = k$.

(b) Write down the system that would need to be solved in order to perform one step of the Implicit method to approximate temperature at time $t = \Delta t = k$.

(5) Write down the two ODE that result from applying the Separation of Variables technique to the Heat Equation. Explain briefly how the boundary conditions affect the form of the ODE.

(6) State the general solution of the Heat Equation, and then give the unique solution corresponding to the IVP defined by the initial temperature distribution $f(x) = \sin \pi x + \frac{1}{100} \sin 10\pi x$.