Q5 661

Work 3 of 6 Problems, clearly marking which problems are to be graded.

(1) Write down the 2-D Heat Equation on the unit square, with boundary and initial conditions. Write down the corresponding general solution using linear combinations of the eigenfunctions $\sin(j\pi x)\sin(k\pi y)$. Write down the exact solution if the initial temperature is the first mode, that is $j = k = 1$ are the only nonzero Fourier coefficient indices.

(2) Write down the 2-D Heat Equation on the unit disk, with boundary and initial conditions. You do not have to write down the corresponding general solution using linear combinations of the eigenfunctions, but do write down the exact solution if the initial temperature is the first mode, that is $m = 0$, $k = 1$ are the only nonzero Fourier coefficient indices. Recall that the eigenfunction $\Psi_{m,j}$ is determined by the Bessel function J_m and involves the j^{th} zero of that function.

(3) Derive the 2×2 linear system for the least-squares fit of a line $y = mx + b$ to a collection of data $\{(x_i, y_i)\}_{i=1,\ldots,n}$ *one of two ways*: By writing down an overdetermined linear system and multiplying both sides by the appropriate matrix, or by defining a quadratic error function and setting its gradient to zero. You do not need to solve the systems for m and b.

(4) Consider doing a Least-Squares fit of functions $y = a2^{bx}$ to the data $\{(0,0), (1,2), (2,4)\}$. Write down the Error function and compute the gradient. Is the resulting system linear or nonlinear? Sketch a few *details* concerning how you would actually solve this system.

(5) State the wave equation in one time variable and one space variable, together with reasonable initial and boundary conditions. Sketch the separation of variables argument very briefly, and write down at least one solution to the boundary value problem (you get to pick the initial value).

(6) Consider an $n = 4$ divisions by $n = 4$ divisions grid of the unit square and functions that are potentially nonzero at the nodes $\{(1,1),(1,2),(2,1),(3,1)\}$ (lying on an L-shape), but zero at the other 21 grid points. Write down the negative Laplacian matrix representing the negative second derivative operator on this L-shaped region.