MAT 238 Fall 2003 Vector functions

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Vector functions

- Notation: $\underline{r} : \mathbf{R} \to \mathbf{R}^3$, $\underline{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$
- Rule of thumb: Everything is done coordinatewise.
- Limit: $\lim_{t\to a} \underline{r}(t) = \langle \lim_{t\to a} r_1(t), \lim_{t\to a} r_2(t) \rangle$
- Derivative: $\underline{r}'(t) = \lim_{h\to 0} \frac{\underline{r}(t+h)-\underline{r}(t)}{h} = \langle r_1'(t), r_2'(t) \rangle$
- Tangent: $\underline{r}'$ is tangent to the curve
- Expected formulas hold: $(\underline{u} \cdot \underline{v})' = \underline{u}' \cdot \underline{v} + \underline{u} \cdot \underline{v}'$ etc.
- Integral: $\int \underline{r}(t) dt = \langle \int r_1(t), \int r_2(t) \rangle$
- Fundamental theorem: $\int_a^b \underline{r}(t) dt = \underline{R}(b) - \underline{R}(a)$ where $\underline{R}' = \underline{r}$
- Indefinite integral: $\int \underline{r}(t) dt = \underline{R}(t) + \underline{C}$, $\underline{C}$ is a vector
- $\underline{r}$ is smooth: if $\underline{r}'$ is continuous and $\underline{r}'$ is never $\underline{0}$

Arc length

- Formula: $\int_a^b |\underline{r}'(t)|\, dt$

Moving objects

- Position: $\underline{r}(t)$
- Velocity: $\underline{v} = \underline{r}'$
- Speed: $s = |\underline{v}|$
- Acceleration: $\underline{a} = \underline{v}' = \underline{r}''$
- Unit tangent vector: $\underline{T} = \frac{\underline{r}'}{|\underline{r}'|}$
- Unit normal vector: $\underline{N} = \frac{\underline{T}'}{|\underline{T}'|}$
- Binormal vector: $\underline{B} = \underline{T} \times \underline{N}$
- Tangent component of $\underline{a}$: $(\underline{T} \cdot \underline{a})\underline{T}$ changes the speed
- Normal component of $\underline{a}$: $\frac{|\underline{r}' \times \underline{r}''|}{|\underline{r}'|}\underline{N}$ changes the direction of velocity
- Newton's law: $\underline{F} = m\underline{a}$