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Double integral of $f : \mathbf{R}^2 \to \mathbf{R}$

- Definition: $\int\int_{[a,b]\times[c,d]} f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=0}^{m} \sum_{j=0}^{n} f(x_i, y_j)\Delta y \Delta x$
- Meaning: Signed volume between the graph of f and the xy-plane
- Fubini's theorem: $\int\int_{[a,b]\times[c,d]} f(x,y)dA = \int_a^b \int_c^d f(x,y)dy\,dx = \int_c^d \int_a^b f(x,y)dx\,dy$
- General region, vertical slices: $\int_a^b \int_{g(x)}^{h(x)} f(x,y)dy\,dx$
- General region, horizontal slices: $\int_a^b \int_{g(y)}^{h(y)} f(x,y)dx\,dy$
- Double integral of constant function: $\int\int_R k dA = k \cdot \text{Area}(R)$
- Integral on a disjoint union: $\int\int_{R_1 \cup R_2} f(x,y)dA = \int\int_{R_1} f(x,y)dA + \int\int_{R_2} f(x,y)dA$
- Mass of plate: $m = \int\int_R \rho(x,y)\,dA$
- Center of gravity of plate: $\bar{y} = \frac{1}{m} \int\int_R y\rho(x,y)\,dA$
- Average value of f on R: $\frac{1}{\text{Area}(R)} \int\int_R f(x,y)\,dA$

Triple integral of $f : \mathbf{R}^3 \to \mathbf{R}$

- Fubini's theorem: $\int\int\int_{[a,b]\times[c,d]\times[m,n]} f(x,y)dV = \int_a^b \int_c^d \int_m^n f(x,y,z)\,dzdy\,dx = \cdots = \int_m^n \int_c^d \int_a^b f(x,y)dx\,dy\,dz$
- Slices perpendicular to the y-axis: $\int_a^b \left(\int\int_{R(y)} f(x,y,z)dA \right) dy$
- Triple integral of constant function: $\int\int\int_B k dV = k \cdot \text{Volume}(B)$
- Mass of body: $m = \int\int\int_B \rho(x,y,z)\,dV$
- Center of gravity of body: $\bar{z} = \frac{1}{m} \int\int\int_B z\rho(x,y,z)\,dV$
- Average value of f on B: $\frac{1}{\text{Volume}(B)} \int\int\int_B f(x,y)\,dV$

Change of variables: $T : Q \to R$

- 2D: $(x,y) = T(u,v)$, $\int\int_R f(x,y)dy\,dx = \int\int_Q f(T(u,v)) \cdot |\det T'(u,v)|\,dv\,du$
- 3D: $(x,y,z) = T(u,v,w)$,
$\int\int\int_R f(x,y,z)dz\,dy\,dx = \int\int\int_Q f(T(u,v,w)) \cdot |\det T'(u,v,w)|\,dw\,dv\,du$
- Polar coordinates: $(x,y) = T(\theta,r) = (r\cos\theta, r\sin\theta)$, $|\det J_{(\theta,r)}T| = r$
- Cylindrical coordinates: $(x,y,z) = T(\theta,r,z) = (r\cos\theta, r\sin\theta, z)$, $|\det T'(\theta,r,z)| = r$
- Spherical cordinates: $(x,y,z) = T(\rho,\theta,\phi) = (\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))$, $|\det T'(\rho,\theta,\phi)| = \rho^2 \sin\phi$