Scalar field: $f: \mathbb{R}^3 \to \mathbb{R}$, Vector field: $\mathbf{E}: \mathbb{R}^3 \to \mathbb{R}^3$

Line integral of scalar fields
- Notation: $\int_C f \, ds$
- Evaluation: $\int_a^b f(r(t))|r'(t)| \, dt$
- Mass of wire: $m = \int_C \rho \, ds$
- Center of gravity of a wire: $\bar{x} = \frac{1}{m} \int_C x \rho \, ds$
- Line integral of constant a function: $\int_C k \, ds = k \cdot \text{Length}(C)$

(Circulation) line integral of vector fields:
- Notation: $\int_C \mathbf{E} \cdot d\mathbf{r} = \int_C \mathbf{E} \cdot \mathbf{T} \, ds$
- Different notation: $\int_C F_1(x, y, z) \, dx + F_2(x, y, z) \, dy + F_3(x, y, z) \, dz$
- Evaluation: $\int_a^b \mathbf{E}(r(t)) \cdot r'(t) \, dt$
- Reversed path: $\int_{C_{\text{rev}}} \mathbf{E} \cdot d\mathbf{r} = - \int_C \mathbf{E} \cdot d\mathbf{r}$
- Application: Work of force field on moving object

Flux line integral (only in 2D, $\mathbf{E}: \mathbb{R}^2 \to \mathbb{R}^2$)
- Notation: $\int_C \mathbf{E} \cdot \mathbf{N} \, ds$
- Evaluation: $\int_a^b \mathbf{E}(r(t)) \cdot r'(t) \perp \, dt$
- Reversed orientation: $\int_C \mathbf{E} \cdot (-\mathbf{N}) \, ds = - \int_C \mathbf{E} \cdot \mathbf{N} \, ds$
- Application: Amount of fluid going through the boundary

Conservative vector fields
- Characterizations:
  (a) $\mathbf{E}$ is conservative ($\int_C \mathbf{E} \cdot d\mathbf{r}$ only depends on the endpoints of $C$)
  (b) $\mathbf{E}$ has a potential $f$ ($\mathbf{E} = \nabla f$)
  (c) $\int_C \mathbf{E} \cdot d\mathbf{r} = 0$ for all simple closed curve $C$
  (d) $\nabla \times \mathbf{E} = 0$ (if $\mathbf{E}$ is nice)
- How to find the potential: $f(x, y, z) = \int_C \mathbf{E} \cdot d\mathbf{r}$ where $C$ is any curve connecting $(0, 0, 0)$ to $(x, y, z)$

Rotation $\equiv$ curl
- Notation: $\text{curl} \mathbf{E} = \text{rot} \mathbf{E} = \nabla \times \mathbf{E}$
- Definition in 2D: $\frac{\partial}{\partial x} F_2(x, y) - \frac{\partial}{\partial y} F_1(x, y)$
- Interpretation in 2D: circulation density
- Definition in 3D: $\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
F_1(x, y, z) & F_2(x, y, z) & F_3(x, y, z)
\end{vmatrix}$
- Interpretation in 3D: length is the maximal circulation density, direction is the direction of the maximum circulation density
Divergence
- Notation: $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$
- Definition: $\frac{\partial F_1(x,y,z)}{\partial x} + \frac{\partial F_2(x,y,z)}{\partial y} + \frac{\partial F_3(x,y,z)}{\partial z}$
- Interpretation: flux density, source or sink

Surface integral of scalar fields
- Notation: $\int \int_S f \, dS$
- Evaluation: $\int \int_R f(r(u,v))|r_u(u,v) \times r_v(u,v)| \, dvdu$
- Mass of aluminum foil: $m = \int \int_S \rho \, dS$
- Center of gravity of aluminum foil: $\bar{y} = \frac{1}{m} \int \int_S y \rho \, dS$
- Surface integral of a constant function: $\int \int_S k \, dS = k \cdot \text{Area } (S)$

Surface (flux) integral of vector fields
- Notation: $\int \int_S \mathbf{F} \cdot d\mathbf{S}$
- Alternative notation: $\int \int_S \mathbf{F} \cdot \mathbf{N} \, dS$
- Evaluation: $\int \int_R \mathbf{F}(r(u,v)) \cdot (r_u(u,v) \times r_v(u,v)) \, dvdu$ or $r_v \times r_u$ depending on the orientation
- Interpretation: stuff going through the surface

Curl theorem
- Interpretation: the integral of circulation density is the circulation on the boundary

Stokes’ theorem
- If $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ and $C$ is the boundary of $S$ ($S$ is on the left of $C$) then $\int_C \mathbf{F} \cdot d\mathbf{r} = \int \int_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$

Green’s theorem (2D version of Stokes’ theorem)
- If $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ and $C$ is the boundary of $R$ ($R$ is on the left of $C$) then $\int_C \mathbf{F} \cdot d\mathbf{r} = \int \int_R \frac{\partial F_2(x,y)}{\partial x} - \frac{\partial F_1(x,y)}{\partial y} \, dydx$

Divergence theorem
- Interpretation: the integral of flux density is the flux on the boundary
- Interpretation: amount coming in and out through the boundary (flux) is equal to the amount created and swallowed (divergence)

Gauss’ theorem
- If $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ and $S$ is the boundary (outward orientation) of the solid $E$ then $\int \int_S \mathbf{F} \cdot d\mathbf{S} = \int \int \int_E \nabla \cdot \mathbf{F} \, dE$

2D version of Gauss’ theorem
- If $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ and $C$ is the boundary of $R$ then $\int_C \mathbf{F} \cdot \mathbf{N} \, ds = \int \int_R \nabla \cdot \mathbf{F} \, dA$