

Turn in your solution to at least two of the problems. Explain your solution in full sentences. Give detailed reasonings. Include diagrams and figures if appropriate.

Problem 1. We have the following information about three different integers:

- their product is an integer;
- their product is a prime;
- one of them is the average of the other two.

What are these numbers? *Hint:* You need to find all such triples and show that there are no others.

Problem 2. There are 30 red, 40 yellow, 50 blue and 60 green balls in a box. We take out balls from the box with closed eyes. On the first turn we take out 1 ball, on the second turn we take out 2, and so on. On the n -th turn we take out n balls. What is the minimum number of balls we need to take out to guarantee the following:

- (1) we have a blue ball;
- (2) we have a red and a green ball;
- (3) we have all four colors.

Problem 3. The front side of a paper square is white while its back side is black. The area of the square is 3 square inches. We folded one of the corners of the paper so that the corner is now on top of the diagonal of the square. Now the area of the black and white regions are the same. How far is the folded corner from the crease line?

Problem 4. How many zeros are there at the end of the product of the numbers $1, 2, 3, \dots, 100$?