299 SPRING 2009 **HOMEWORK 12**

Turn in your solution to at least four of the problems. **Explain your solution in full sentences. Give detailed reasonings.** Include diagrams and figures if appropriate.

Definition. A *stone distribution* is a placement of stones on the vertices of a graph G. Each vertex has non-negative integer number of stones. A *move* consists of two steps:

- remove two stones from a vertex;
- place one stone on an adjacent vertex.

Given a stone distribution on a graph G, and a root vertex r of G we try to place a stone on r using a sequence of moves. We define the *stone number* $\pi(G)$ to be the minimum number of stones such that it is possible to move a stone to any root vertex from any initial stone distribution of $\pi(G)$ stones.

Problem 1. Find the stone number of a path containing 3 vertices. Find a conjecture for a path with n vertices?

Definition. The *distance* between two vertices of a graph is the length of the shortest path between them. The *diameter* of a graph is the largest distance between any two vertices.

Problem 2. Find the stone number of a graph of diameter 1.

Problem 3. Find the stone number of a cycle of length 5.

Problem 4. Find the stone number of all trees containing 5 vertices.

Problem 5. Find a lower bound for the stone number of a graph in terms of the number of vertices of the graph.

Problem 6. Find a lower bound for the stone number of a graph in terms of the diameter of the graph.