MAT 362 Spring 2007 12. Programming assigment: Finding eigenvalues (Wave) Name:

Instructor: Nándor Sieben
e-mail: nandor.sieben@nau.edu
http://jan.ucc.nau.edu/∼ns46/362/362.html
Office: AMB 175
Hours: MW 10:10-12:10, F 10:10-11:10

Find the first four smallest absolute value eigenvalues and corresponding eigenfunctions of the tridiagonal matrix

$$\begin{pmatrix} -2 & 1 & 0 & \cdots & & 0 \\ 1 & -2 & 1 & & & 0 \\ 0 & 1 & -2 & & & 0 \\ \vdots & & & & & \vdots \\ 0 & & & & -2 & 1 \\ 0 & \cdots & & & 1 & -2 \end{pmatrix}$$

representing the second derivative. Use a matrix with 100 rows and columns.
Here is a possible approach. Write a program to create this matrix and writes it into the `wave.in` file. Write another program called `shift` that has a command line argument s, reads a matrix A from standard input and writes the matrix $A - sI$ to standard output. Use your inverse power method with the matrix $A - sI$. Get the last line of the output using the `tail` command and feed this last line to another program called `getvector` that has the command line argument s, swallows the number of iterations, writes the eigenvalue added to s to standard error `cerr` and the eigenvector to standard output as a single column. The supplied script called `go` shows how to combine these pieces into a single command.

Turn in:

- This problem sheet with your name.
- A summary sheet explaining what you did, how you approached the problem, what was accomplished, what was not accomplished, etc.
- A list of the found eigenvalues.
- The plots of the corresponding eigenfunctions.

Website:

- Create a directory called `12wave` on your web site and make all your input, output and source files available in this directory. Write the url for the website on this problem sheet.

Hints:

- The eigenvalues you are looking for are small negative numbers.
- Use a tolerance of 10^{-15} in your inverse power method.
- You can write the different pieces of code in different directories and collect the executables in a main working directory where you run your experiments.
- Test the different pieces of codes separately before you combine them into a single package.
- You should be able to recognize the eigenfunctions on the plots and decide if you skipped any eigenvalues.
- Modify the command line argument example code together with your matrix library to write `shift`.
- Don't forget that you need to shift the eigenvalues of $A - sI$ by s to get the eigenvalues of A.
- The reason we want to write the eigenvalue to standard error is that standard error is not redirected to a file even if standard output is. This way we can see the eigenvalue and write the eigenvector to a file for gnuplot at the same time.
- If a gnuplot input file contains only a single column, then gnuplot uses these values as y-values together with $1, 2, \ldots$ as x-values.
- It is possible to find the exact eigenvalues for the wave equation. Can we use these values to predict the approximate eigenvalues of our matrix? How does it depend on the size of the matrix?