
Write a program that solves the boundary value problem

$$y''(x) = y'(x) + 2y(x) + \cos(x), \quad 0 \leq x \leq \pi/2, \quad y(0) = -0.3, \quad y(\pi/2) = -0.1$$

using finite differencing with n points. Run your code with $n = 10$. Print the augmented matrix of the linear system and plot the solution.

Turn in:

- This problem sheet with your name.
- A summary sheet explaining what you did, how you approached the problem, what was accomplished, what was not accomplished, etc.
- The augmented matrix of the system of linear equations.
- The plot of the solution.

Website:

- Create a directory called **13bvp** on your web site and make all your input, output and source files available in this directory. Write the url for the website on this problem sheet.

Input:

The problem is hardwired into the code.

Output:

The augmented matrix to standard error. The (x, y) coordinates to standard output.

Sample output: $n = 3$

Augmented matrix:

-14.9691 5.21132 0 3.25122

7.7578 -14.9691 5.21132 0.707107

0 7.7578 -14.9691 0.903815

Output:

0 -0.3

0.392699 -0.315685

0.785398 -0.282906

1.1781 -0.206996

1.5708 -0.1

Hints:

- Build the augmented matrix and use your old linear system solver code to find (y_1, \dots, y_n) .
- Try to be as generic as possible. Set the variables a, b, y_0, y_{n+1}, n and possibly even the p, q, r functions as well and calculate the rest in terms of these. This makes it a lot easier to solve other problems with minimal modification.
- Verify your code against the sample output with $n = 3$.
- Do not forget to print the (x_0, y_0) and (x_{n+1}, y_{n+1}) pairs.
- $\pi/2 = \arcsin(1)$. In C++ it is called **asin**.
- How could we find the values of the solution at input values which are not grid points?