MAT 362 Spring 2007 6. Programming assignment: Steepest descent Name:

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Hours: MW 10:10-12:10, F 10:10-11:10

Write a program implementing the steepest descent method. Use your program to find solutions for the system:

$$3x^2 - y^2 = 0$$
$$3xy^2 - x^3 - 1 = 0$$

At each iteration print the number of iterations and the estimate for the root. Do not forget to print the initial guess as iteration 0. Modify the supplied gnuplot file `steepestex.gnu` to create a contourplot shoving the path of the estimated root through the iterations (see `steepestex.ps`). First, use the initial guess $(-1, 1.5)$, then use the initial guess $(-1, 2)$. Then find a third initial guess that leads to a different solution.

Turn in:

- This problem sheet with your name.
- A summary sheet explaining what you did, how you approached the problem, what was accomplished, what was not accomplished, etc. Include a table showing the three roots you have found and the number of iterations required.
- Three countourplots for the three initial guesses.

Website:

- Create a directory called `6steepest` on your web site and make all your input, output and source files available in this directory. Write the url for the website on this problem sheet.

Hints:

- Start with `minimum.cc` which is the one dimensional version of the code you need.
- You need to solve a system of nonlinear equations with two unknowns and two equations. This means you are looking for the root of a function $f : \mathbf{R}^2 \to \mathbf{R}^2$. The first step is to define $g : \mathbf{R}^2 \to \mathbf{R}$ by taking the norm of f. Then a root of f becomes a minimum of g.
- When you define a function that implements $g : \mathbf{R}^2 \to \mathbf{R}$, make the input variable have type `Trow`. The function should return type `double`. The gradient function takes an input of type `Trow` and returns type `Trow`.
- The gradient gives the direction of the greatest increase. The direction of the greatest decrease is the opposite of the gradient.
- Use `matrix.cc` and `matrix.h` to implement generic matrix operations (scalar multiple, sum, norm printing, reading).
- To simplify your work, write a go script that compiles and runs your code and runs gnuplot.
- Use a tolerance of 10^{-8}.
- Gnuplot picks the viewing window automatically based on the data points. This usually results in different units on the two axes. This is not desirable in this assignment because the angles are distorted. To see that the path of the estimated root is perpendicular two the level curves, you need to use the same units on the axes. This is achieved by the `set size ratio -1` command. The postscript terminal uses landscape mode. You may want to use the postscript portrait terminal with fixed ratios.
- The creation of a contourplot involves two steps. First a data file is created for the contours. This data file is used in the actual plot. Take a look at the created data file and read the gnuplot documentation to understand the details.
- Steepest descent converges slowly close to the solution but it does a good job to get relatively close to the solution. This is the reason it is often used as a first step to get an initial guess to start Newton's method which needs an initial guess relatively close to the solution.