

1. Consider the boundary value problem $2xy''(x) - y'(x) + y(x) = x^2 + 3$ where $-2 \leq x \leq 2$, $y(-2) = 9$ and $y(2) = 1$. Use the differencing method with 3 division points to find an approximate solution.

Solution:

$$\begin{aligned}
 & 2 \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{1}{1^2} \left(\begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \\ 1 \end{pmatrix} \right) \\
 & - \frac{1}{2 \cdot 1} \left(\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} -9 \\ 0 \\ 1 \end{pmatrix} \right) + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} (-1)^2 + 3 \\ 0^2 + 3 \\ 1^2 + 3 \end{pmatrix} \\
 & \left(\begin{pmatrix} 4 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & -4 \end{pmatrix} - \begin{pmatrix} 0 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -18 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -9/2 \\ 0 \\ 1/2 \end{pmatrix} \\
 & \begin{pmatrix} 5 & -5/2 & 0 \\ 1/2 & 1 & -1/2 \\ 0 & 5/2 & -3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \frac{35}{2} \\ 3 \\ \frac{5}{2} \end{pmatrix}
 \end{aligned}$$

The solution is $(y_1, y_2, y_3) = (4, 1, 0)$. The exact solution can be guessed, it is $y(x) = (x - 1)^2$.