MAT 365 Fall 2002 Test 3 Name:
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Hours: MWF 11:20 – 12:20 MW 1:40 – 2:40

Show your work neatly and explain how you get your answers in full sentences. Explain what results are used from the lectures or from the book. Do not turn in irrelevant calculations and scratch work. The credit you receive depends on how well I can follow your reasoning. An answer alone will not receive credit. Draw a figure large enough to show your notation clearly. Start each problem on a new page.

1. Show that the incenter I, the center of gravity G and the Nagel point M of $\triangle ABC$ are collinear and G trisects IM closer to I. Hint: show that $\underline{g} + \frac{1}{2}\overrightarrow{MG} = \underline{i}$.

2. Let AX be a Cevian of $\triangle ABC$ with length p, dividing BC into segments $BX = m$ and $XC = n$. Show that $a(p^2 + mn) = b^2m + c^2n$. Hint: Use the law of cosines for $\angle BXA$ and $\angle CXA$.

3. A generalized altitude of a tetrahedron is a plane that contains the midpoint of an edge and is perpendicular to the opposite edge. Let O be the center of the circumsphere and G be the center of gravity of the tetrahedron $ABCD$. Let T be the mirror image of O with respect to G. Show that the generalized altitudes contain T. Hint: follow the proof of the Euler line to show that $2\overrightarrow{OG}$ points to the generalized altitudes.
Note 1: OT is called the *Euler line of the general tetrahedron.*
Note 2: The altitudes of a tetrahedron are not concurrent in general.
Note 3: There is a *Feuerbach sphere* of the general tetrahedron whose center trisects OT closer to T and whose radius is $\frac{1}{3}$ of the radius of the circumsphere.

4. Let the external bisectors of $\triangle ABC$ intersect the opposite sides at points X, Y and Z. Show that X,Y and Z are collinear.

5. Recall that the opposite edges of an orthocentric tetrahedron are perpendicular. Let $ABCD$ be an orthocentric tetrahedron. Show that $AB^2 + CD^2 = AD^2 + BC^2$.

6. The reflection s_a of the median m_a with respect to the angle bisector e_a is called a *symmedian* of $\triangle ABC$. Prove that the symmedians are concurrent. The point of concurrency is called the symmedian point.