

1. Show that in  $\triangle ABC$  we have  $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$ .

*Solution:* If  $K = A(ABC)$  then we have

$$\begin{aligned}\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} &= \frac{a}{2K} + \frac{b}{2K} + \frac{c}{2K} \\ &= \frac{s}{K} \\ &= \frac{1}{r}.\end{aligned}$$

2. Let  $P$  be a point inside the regular  $\triangle ABC$ . Show that if  $x, y, z$  are the distances of  $P$  from  $a, b$  and  $c$  respectively then  $x + y + z = h$  where  $h$  is the height of the triangle.

*Solution:* We have

$$\begin{aligned}ah &= A(ABC) \\ &= A(ABP) + A(BCP) + A(CAP) \\ &= ax + ay + az \\ &= a(x + y + z)\end{aligned}$$

and so  $h = x + y + z$ .