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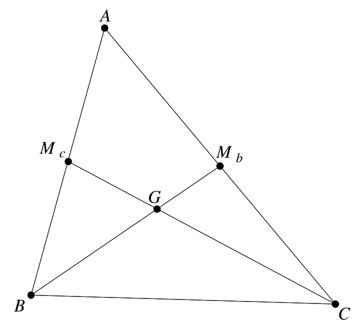
Hours: MWF 11:20 – 12:20 MW 1:40 – 2:40

1. Show that in $\triangle ABC$ we have

$$\frac{3s}{2} \le m_a + m_b + m_c \le 2s.$$

Hint: use the $\triangle$ inequality.

Solution: Use the notation of the figure.



The $\triangle$ inequality for $\triangle GM_bM_c$ gives

$$\frac{1}{3}m_b + \frac{1}{3}m_c \ge \frac{a}{2}.$$

Similarly we can get

$$\frac{1}{3}m_a + \frac{1}{3}m_c \ge \frac{b}{2}$$
$$\frac{1}{3}m_a + \frac{1}{3}m_a \ge \frac{c}{2}.$$

Adding the inequalities gives

$$m_a + m_b + m_c \ge \frac{3s}{2}.$$

The $\triangle$ inequality for $\triangle M_bM_cV$ gives

$$m_c \le \frac{a}{2} + \frac{b}{2}.$$

Similarly we can get

$$m_a \le \frac{b}{2} + \frac{c}{2}$$
$$m_b \le \frac{a}{2} + \frac{c}{2}.$$

Adding the inequalities gives

$$m_a + m_b + m_c \le 2s.$$