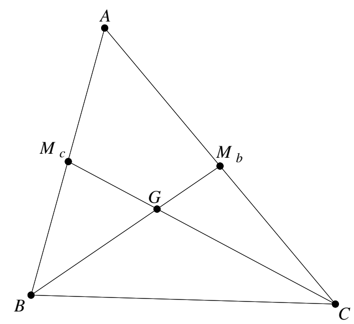
Instructor: Nándor Sieben
e-mail: nandor.sieben@nau.edu
http://jan.ucc.nau.edu/∼ns46/365/365.html
Office: AMB 175, Phone: 523-6874
Hours: MWF 11:20 – 12:20 MW 1:40 – 2:40

1. Show that in $\triangle ABC$ we have

$$\frac{3s}{2} \leq m_a + m_b + m_c \leq 2s.$$

Hint: use the $\triangle$ inequality.

Solution: Use the notation of the figure.



The $\triangle$ inequality for $\triangle GM_bM_c$ gives

$$\frac{1}{3}m_b + \frac{1}{3}m_c \geq \frac{a}{2}.$$

Similarly we can get

$$\frac{1}{3}m_a + \frac{1}{3}m_c \geq \frac{b}{2}$$
$$\frac{1}{3}m_a + \frac{1}{3}m_a \geq \frac{c}{2}.$$

Adding the inequalities gives

$$m_a + m_b + m_c \geq \frac{3s}{2}.$$

The $\triangle$ inequality for $\triangle M_bM_cV$ gives

$$m_c \leq \frac{a}{2} + \frac{b}{2}.$$

Similarly we can get

$$m_a \leq \frac{b}{2} + \frac{c}{2}$$
$$m_b \leq \frac{a}{2} + \frac{c}{2}.$$

Adding the inequalities gives

$$m_a + m_b + m_c \leq 2s.$$