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Hours: MWF 11:20 – 12:20 MW 1:40 – 2:40

1. Let $ABCD$ a quadrilateral. Let X be the trisecting point of AB closer to A. Let Y be the midpoint of BC and Z be the midpoint of CD. Let W be the trisecting point of AD closer to A. Find the position vector of the intersection P of XZ and YW in terms of the position vectors of A, B, C and D. In what ratio does P divide XZ?

Solution: We have

$$\begin{aligned} \underline{x} &= \frac{2\underline{a}+\underline{b}}{3} \\ \underline{y} &= \frac{\underline{b}+\underline{c}}{2} \\ \underline{z} &= \frac{\underline{c}+\underline{d}}{2} \\ \underline{w} &= \frac{\underline{d}+2\underline{a}}{3}. \end{aligned}$$

Point P must satisfy

$$\alpha\underline{x} + (1-\alpha)\underline{z} = \underline{p} = \beta\underline{y} + (1-\beta)\underline{w}.$$

and so

$$\frac{2}{3}\alpha\underline{a} + \frac{\alpha}{3}\underline{b} + \frac{1-\alpha}{2}\underline{c} + \frac{1-\beta}{2}\underline{d} = \frac{2(1-\beta)}{3}\underline{a} + \frac{\beta}{2}\underline{b} + \frac{\beta}{2}\underline{c} + \frac{1-\beta}{3}\underline{d}.$$

Comparing coeeficients gives

$$\begin{aligned} \frac{\alpha}{3} &= \frac{\beta}{2} \\ \frac{1-\alpha}{2} &= \frac{\beta}{2} \end{aligned}$$

and so $\alpha = \frac{3}{5}$ and $\beta = \frac{2}{5}$. Thus P divides XZ in the ratio of $\frac{1-\alpha}{\alpha} = \frac{2}{3}$.

2. Let M be the intersection point of the perpendicular chords AB and CD of circle k whose center is O. Show that $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 2\overrightarrow{OM}$.

Solution: Let

$$\overrightarrow{OX} = \frac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 2\overrightarrow{OM}}{2} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2} + \frac{\overrightarrow{OC} + \overrightarrow{OD}}{2}.$$

The vector $\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}$ points to the midpoint of AB and $\frac{\overrightarrow{OC}+\overrightarrow{OD}}{2}$ is parallel to AB so X is on AB. Similarly X is on CD and so we must have $X = M$.