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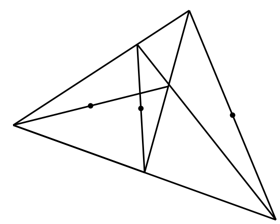
Hours: MWF 11:20 – 12:20 MW 1:40 – 2:40

1. a. Let $\underline{a} = \overrightarrow{OA}$, $\underline{b} = \overrightarrow{OB}$, $\lambda\underline{a} = \overrightarrow{OA'}$ and $\mu\underline{b} = \overrightarrow{OB'}$. Let V be the intersection of AB' and $A'B$. Express $\underline{v}$ in terms of $\underline{a}$, $\underline{b}$, λ and μ.

b. Let X, Y and Z be the midpoints of OV, AB and $A'B'$ respectively. Show that X, Y and Z are collinear. Hint: show that $\underline{x} - \underline{z} = \lambda\mu(\underline{x} - \underline{y})$.



Solution: a. Using the usual approach of divisions and comparison of coefficients we get

$$\underline{v} = \frac{\lambda(1-\mu)}{1-\lambda\mu}\underline{a} + \frac{\mu(1-\lambda)}{1-\lambda m}\underline{b}.$$

b. We have

$$\underline{x} = \frac{1}{2}\underline{v}, \quad \underline{y} = \frac{\underline{a}+\underline{b}}{2}, \quad \underline{z} = \frac{\lambda\underline{a}+\mu\underline{b}}{2}$$

and so after a little bit of calculation we get

$$\underline{x} - \underline{y} = \frac{1}{2}\frac{\lambda-1}{1-\lambda\mu}\underline{a} + \frac{1}{2}\frac{\mu-1}{1-\lambda\mu}\underline{b}$$

$$\underline{x} - \underline{z} = \frac{1}{2}\frac{\lambda-1}{1-\lambda\mu}\lambda\mu\underline{a} + \frac{1}{2}\frac{\mu-1}{1-\lambda\mu}\lambda\mu\underline{b}.$$