
1. Let E , F and G be the points where the inscribed circle of $\triangle ABC$ touches sides a , b and c respectively. Show that AE , BF and CG are concurrent. The common point is called the Gergonne point.

Solution: Since $AF = AG$, $BG = BE$ and $CE = CF$ we have

$$\frac{AG}{GB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FA} = \frac{AG}{BE} \cdot \frac{BE}{CF} \cdot \frac{CF}{AG} = 1$$

and so the result follows from the converse of Ceva's theorem.

2. Show that the reflection H'_a of the orthocenter H of $\triangle ABC$ with respect to side a lies on the circumcircle.

Solution: We know that the reflection H_a of H with respect to the midpoint M_a of BC is on the circumcircle. H'_a is the reflection of H_a with respect to the perpendicular bisector l of BC and so H'_a is on the circumcircle since the circumcircle is symmetric with respect to l .