

1. Let P and Q be projections. Show that $P + Q$ is a projection if and only if $PQ = 0 = QP$.
2. Let $L : \mathbf{P}_3[x] \rightarrow \mathbf{R}^2$ defined by $L(p(x)) = (p''(0), p'(0) + p(0))$. Let $B = \{x^2 - 1, 1 + x, 2 - x\}$ and $C = \{(1, 2), (3, 4)\}$ be ordered bases for \mathbf{P}_3 and \mathbf{R}^2 respectively. Find $[L]_B^C$ and $[L(1-x)]_C$.
3. Show that if V is a finite dimensional vector space and $L : V \rightarrow V$ is a linear transformation then $\text{rank}(L^k) = \text{rank}(L^{k+1})$ for some k and $V = \ker(L^k) \oplus \text{im}(L^k)$.