MAT 516 Spring 2009 Final (take home part) Name:

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You can use all results that were proved in class, in the book or was assigned as homework. All problems have equal credit. Since this is a test, you are not allowed to discuss it with each other.

Problem	1	2	3	Overall
Score				

1.
a. Show that similar matrices have equal traces.
b. Show that if A is a complex matrix such that $A^2 = A$ then $\text{trace}A = \text{rank}A$.

2. Recall that $\mathcal{L}(U, V)$ is the vector space of linear transformations from U to V.
Let V be a vector space over the field $\mathbf{F}$. The *dual space* of V is $V^* = \mathcal{L}(V, \mathbf{F})$. If $S \subseteq V$ then the *annihilator* of S is $S^0 = \{f \in V^* \mid f(S) = \{\underline{0}\}\}$.
a. Show that S^0 is a subspace of V^*.
b. Show that if W_1 and W_2 are subspaces of V then $W_1^0 = W_2^0$ implies $W_1 = W_2$.
c. Show that if W is a subspace of V then $\text{dim}W^0 = \text{dim}V - \text{dim}W$. Hint: note that $L \mapsto [L]_B^D : \mathcal{L}(U, V) \to \mathbf{F}^{m \times n}$ is an isomorphism.

3. Let $L : U \to V$ be a linear transformation between inner product spaces. A function $L^* : V \to U$ is called an *adjoint* of L if $\langle Lx, y\rangle_V = \langle x, L^*y\rangle_U$ for all $x \in U$ and $y \in V$. Show that if U and V are finite dimensional then
a. L^* is linear;
b. $(\text{im}L^*)^\perp = \text{ker}L$.