

1. Use a combinatorial argument to show that

a)

$$\sum_{k=0}^n \binom{a}{k} \binom{b}{n-k} = \binom{a+b}{n}$$

b)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

2. Let n be a nonnegative integer and $x = \sqrt{238747}$. Prove that the arithmetic mean of $(1+x)^n$ and $(1-x)^n$ is an integer.

3. Assume that every number of the N -th row of Pascal's triangle is odd. What can we say about N ? Hint: show and use the fact that $\binom{N}{2^l} = \frac{N-2^l+1}{2^l} \binom{N}{2^l-1}$.