
1. Show that every k -regular bipartite graph has a perfect matching, that is a matching that saturates every vertex.

Definition: An $m \times n$ latin rectangle is an $m \times n$ matrix $m_{i,j}$ with integer entries satisfying

i) $1 \leq m_{i,j} \leq n$;

ii) no two entries in any row or in any column are equal.

2. Show that any latin rectangle can be extended to a latin square by adding $n - m$ extra rows. Hint: use Hall's theorem and pigeon hole to add one extra row.

3. Let H be a hypergraph with vertex set V and edge set E . In an achievement game played on H , two players alternately mark one vertex using their own colors. A player wins by marking all vertices of any the edge.

Suppose that every edge contains at least a vertices and every vertex is in at most b edges. For example in Tic-Tac-Toe we have $a = 3$ and $b = 4$. Show that the second player can force a draw if $a \geq 2b$. Hint: use a matching on an appropriate bipartite graph