

1. Let  $G$  be a simple graph with  $2n$  vertices. Show that if the degree of every vertex is at least  $n$  then  $G$  has a perfect matching. Hint: show that a maximum matching must saturate every vertex otherwise there would be an alternating path
2. Show that Hall's theorem is not true for infinite bipartite graphs.
3. Determine the number of perfect matchings for the following graphs.
  - a.  $K_{n,n}$
  - b.  $K_n$
  - c.  $P_n \square P_2$