

1. Find a close form for

a.

$$\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \cdots + \binom{n}{k}$$

Hint: use Pascal's triangle to visualize these numbers

b.

$$\sum_{k=m}^n \binom{k}{m} \binom{n}{k}$$

Hint: try to eliminate k from one of the binomial coefficients.

2. How many words can we make using the letter A exactly m times and the letter B at most n times. Find the answer in a closed form.

3. Two candidates are running for president. The winner received n votes while the loser received m votes. As the votes are counted one by one, the counters announce who is ahead. What is the probability that the winner is ahead at each stage?