

1. Determine the number  $a_n$  of  $n$ -digit numbers with each digit odd, where the digits 1 and 3 occur an even number of times.
2. Show that the generating function of the Stirling number sequence  $S(0, k), S(1, k), S(2, k) \dots$  is

$$g_k(x) = \frac{x^k}{(1-x)(1-2x)(1-3x)\cdots(1-kx)}.$$

Hint: Use the recursive formula. Build  $g_k(x)$  from  $g_{k-1}(x)$ .

3. Use the generating function  $g_k(x)$  found in Problem 2 to
  - a. get a formula for  $S(m, k)$ . Hint: Use partial fractions.
  - b. show that  $S(n, k) \equiv_2 \binom{n-j-1}{n-k}$  where  $j = \lfloor k/2 \rfloor$ .