

- 1.a Use the Cauchy-Frobenius-Burnside theorem to find the number of simple graphs on 4 vertices.
- b. Draw a picture of these graphs.

**Definition:** A group  $G$  acts *faithfully* on the set  $X$  if for all  $g \neq h$  the functions  $x \mapsto g \cdot x$  and  $x \mapsto h \cdot x$  are different.

2. Let  $G$  act on  $X$ .
  - a. Show that the set  $F$  of all elements of  $G$  fixing every element of  $X$  is a subgroup of  $G$ .
  - b. Show that  $G$  acts faithfully on  $X$  if and only if  $F$  contains only the identity.
  - c. Show that the quotient group  $G/F$  acts faithfully on  $X$ . Hint: You need to define the operation  $G/F \times X \rightarrow X$ , show that the operation is well defined, show that it is an action, show that it is faithful.

3.a Use Pólya counting to find the number of nonequivalent  $1 \times 2 \times 3$  bricks built from 2 yellow and 4 pink cubes.

- b. Draw a picture of these bricks.