# Filters, isometries, and wavelet representations of the Baumslag-Solitar group

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We consider filter banks associated to the dilation $x \to Nx$ on $\mathbb{R}$, where $N$ be a positive integer greater than 1. These consist of a family of Borel functions $m_i : \mathbb{T} \to \mathbb{C},\ 0\ \leq i\ \leq N-1$ satisfying

$$\sum_{k=0}^{N-1} m_i(ze^{\frac{2\pi ik}{N}})\overline{m_j(ze^{\frac{2\pi ik}{N}})} \ = \ N\delta_{i,j},\ \text{a.e}\ z \in \mathbb{T}.$$

Usually, but not always, one wants $m_0(1) = \sqrt{N},\ m_0$ Lipschitz at 1 and non-vanishing in a large enough neighborhood of 1. In 1997 O. Bratteli and P. Jorgensen showed that defining operators $\{S_i :\ 0 \leq i \leq N-1\}$ on $L^2(\mathbb{T})$ by

$$S_i(f)(z) \ = \ m_i(z)f(z^N),\ 0 \leq i \leq N-1,$$

the family $\{S_i\}$ are isometries and give a representation of the Cuntz algebra $\mathcal{O}_N$ on $L^2(\mathbb{T})$. This talk will discuss to what extent one can relax the conditions on the filters $\{m_i\}$ and still come up with generalized filter banks that give rise to pure isometries on more general Hilbert spaces, and what sort of relations the isometries satisfy. These isometries can be used to construct directly a variety of generalized multiresolution analyses in wavelet and frame theory.

At the same time, one can use these filters to construct representations of the Baumslag-Solitar group $BS_N$, that is, the group with two generators $a$ and $b$ satisfying the single relation $aba^{-1} = b^N$. We discuss what knowledge can be gleaned about these representations from the filter banks.

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