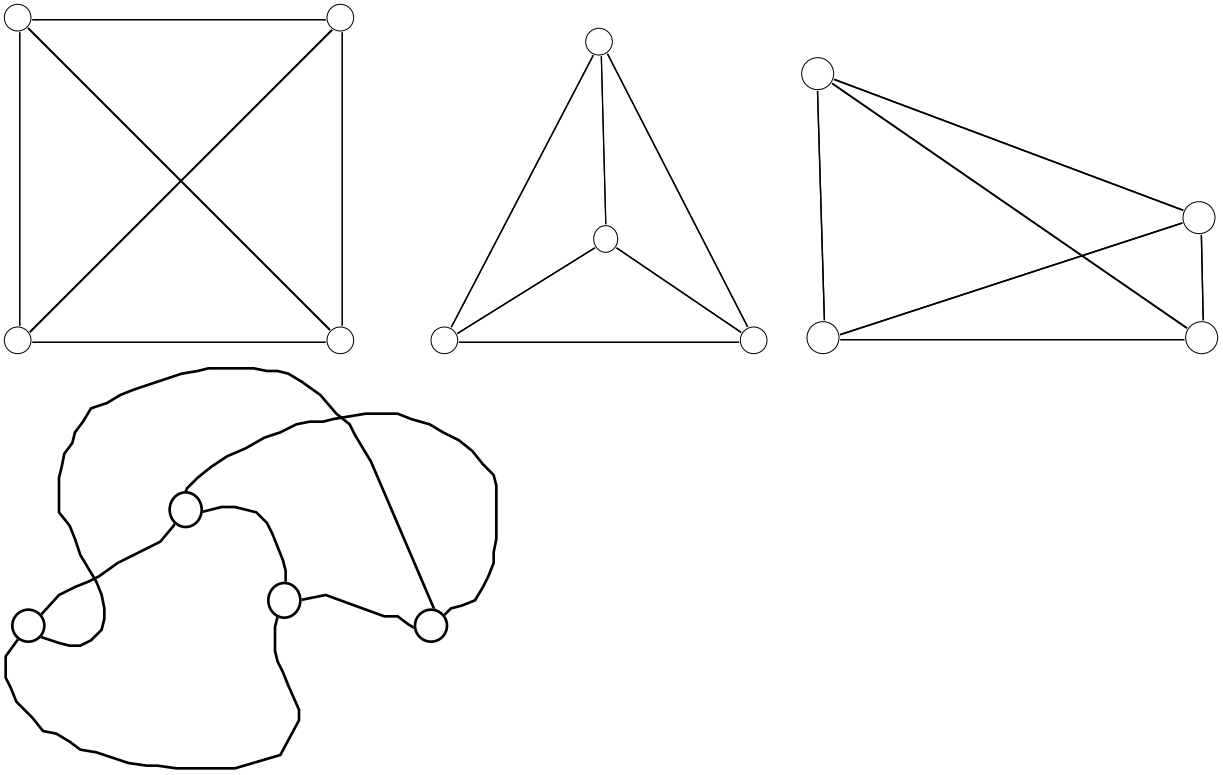
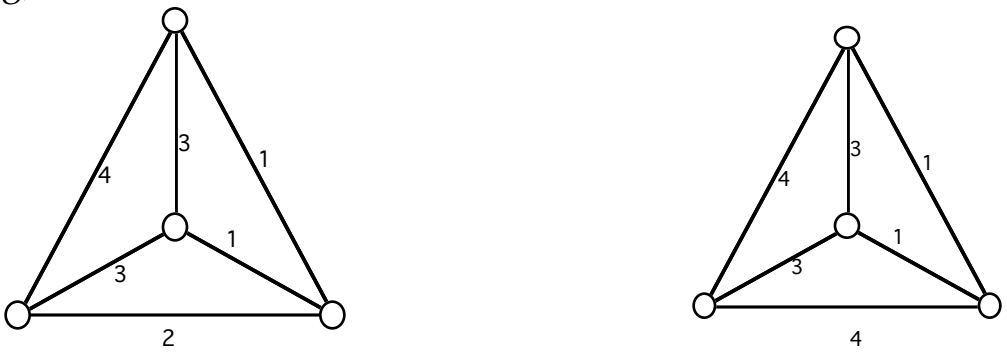
Sub-Dominant k-Colorings of Complete Graphs

The complete graph on N vertices, called K_N, is just N points called vertices (one vertex, two vertices), each and every pair joined by an edge, which might be straight, bowed, crooked, dotted, dashed, blue or chocolate. For instance, each of these is a picture of K_4:

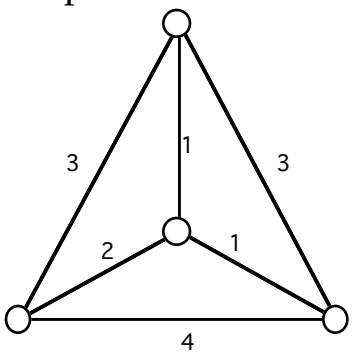


A k-coloring of K_N is any way of labelling the edges with numbers 1, 2, 3, 4, ... , k, using at least one of each. For instance the left diagram below is a 4-coloring of K_4, while the right one is not (no 2's showing).

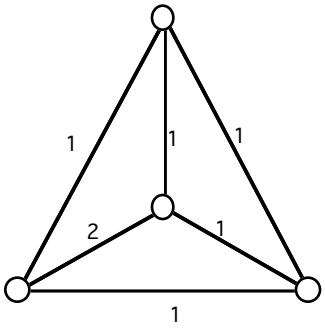


a 4-coloring not a coloring

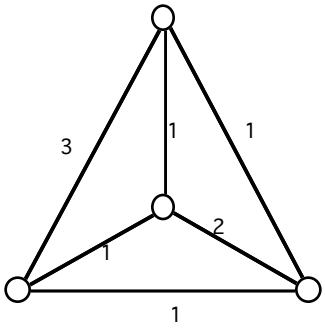
Changing the picture around does not change the coloring.. For example, the picture below is essentially the same as the diagram on the left above.



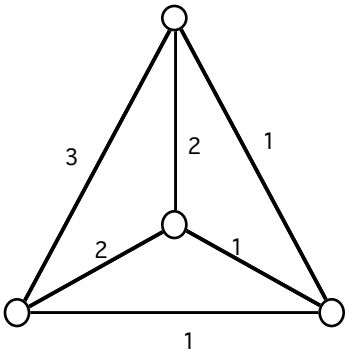
Now, a k-coloring is sub-dominant provided that in every triangle, either all three labels are the same or the smaller one occurs twice. Say that another way: in any triangle, if two of the labels are different, then the third label is equal to the smaller of the first two. Here are some subdominant colorings of K4:



2-coloring



3-coloring



3-coloring

Homework: determine all subdominant colorings of K5.