STA 675 – St. Laurent
Spring 2006

Assignment # 8

Due: 27 April 2006

1. Let **X** be distributed $N_3(\underline{\mu}, \Sigma)$ with density proportional to $\exp(-Q/2)$ where
$$Q = \frac{3}{2}x_1^2 + 2x_2^2 + x_3^2 - 3x_1x_2 + 2x_1x_3 - 2x_2x_3 + 10x_1 - 14x_2 + 8x_3 + 26$$
Find (a) Σ^{-1}; (b) Σ; and (c) $\underline{\mu}$.

2. (Problem from class on 20 April 2006): Let **Y** be distributed $N_n(\mathbf{0}, \mathbf{I}_n)$ and **A** be an orthogonal projection matrix of rank *p*. Show that $\mathbf{Y}^\mathsf{T}\mathbf{A}\mathbf{Y}$ has a $\chi^2_{(p)}$ distribution.

3. Let $\mathbf{X} \sim N_2(\underline{\mu}, \Sigma)$ where $\mathbf{X} = (X_1,\ X_2)^\mathsf{T}$, $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$, $\sigma_1 > 0$, $\sigma_2 > 0$.
   (a) What conditions on ρ are necessary in order to insure that Σ will be positive definite?
   Let $\mathbf{Y} = (X_1 + X_2,\ X_1 - X_2)^\mathsf{T}$.
   (b) Write **Y** as a linear transformation of **X**. What is the distribution of **Y**?
   (c) What conditions on $\underline{\mu}$ and/or Σ are necessary in order for $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$ to be independent?

4. Problem 5.27 on page 193 of the textbook.

5. Let $\mathbf{X} \sim N_n(\mathbf{0}, \Sigma)$ where Σ is positive definite. Show that $\mathbf{X}^\mathsf{T}\Sigma^{-1}\mathbf{X} \sim \chi^2_{(n)}$.