

# **The Operational Sex Ratio, the Potential Reproductive Rate, and the Opportunity for Sexual Selection**

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# Objectives

1. Definitions of OSR/PRR.
2. What do they *really* measure?
3. Do better measures exist?

# What Is The Definition of OSR?

Originally defined by S.T. Emlen 1976 as: “*the ratio of potentially receptive males to receptive females at any time.*”

A measure of  
the *level of  
competition*  
for mates,  
and therefore,  
of the  
*intensity of  
sexual  
selection.*



# The Operational Sex Ratio

Emlen & Oring 1977

$$OSR = \frac{N_{\text{mature males}}}{N_{\text{receptive females}}}$$

**A reproductive competition coefficient.**

$OSR > 1$  = females are rare,  
competition for mates is  
intense.

$OSR < 1$  = females are  
abundant, competition for  
mates is relaxed.





## However,

This definition has ***not remained consistent*** among researchers.

### **3 primary variants of the original concept of OSR:**

1. Changes in the ***ratio*** itself.
2. Changes in how individuals ***are (or are not)*** included in estimates.
3. Changes that attempt to incorporate the effects of ***parental investment***.

**OSR is widely presumed to predict/induce evolutionary responses by populations**

# Changes In Ratio

## The “Operational Sex Ratio” has been expressed in several ways

Ratio of potentially receptive males to females at any time (Emlen 1976)

Ratio of sexually active males to fertilizable females at any time

(Emlen & Oring 1977, and most others).

Reciprocal of the sex ratio ( $R_O = 1/R = N_{males}/N_{females}$ ; Shuster & Wade 2003)

Number of males and females ready to mate (Nyman et al. 2006)

Ratio of matured females to males (Yamamoto & Edo 2006)

Fertilizable females to sexually active males at any given time

(Forbes et al. 2005; Prohl 2006).

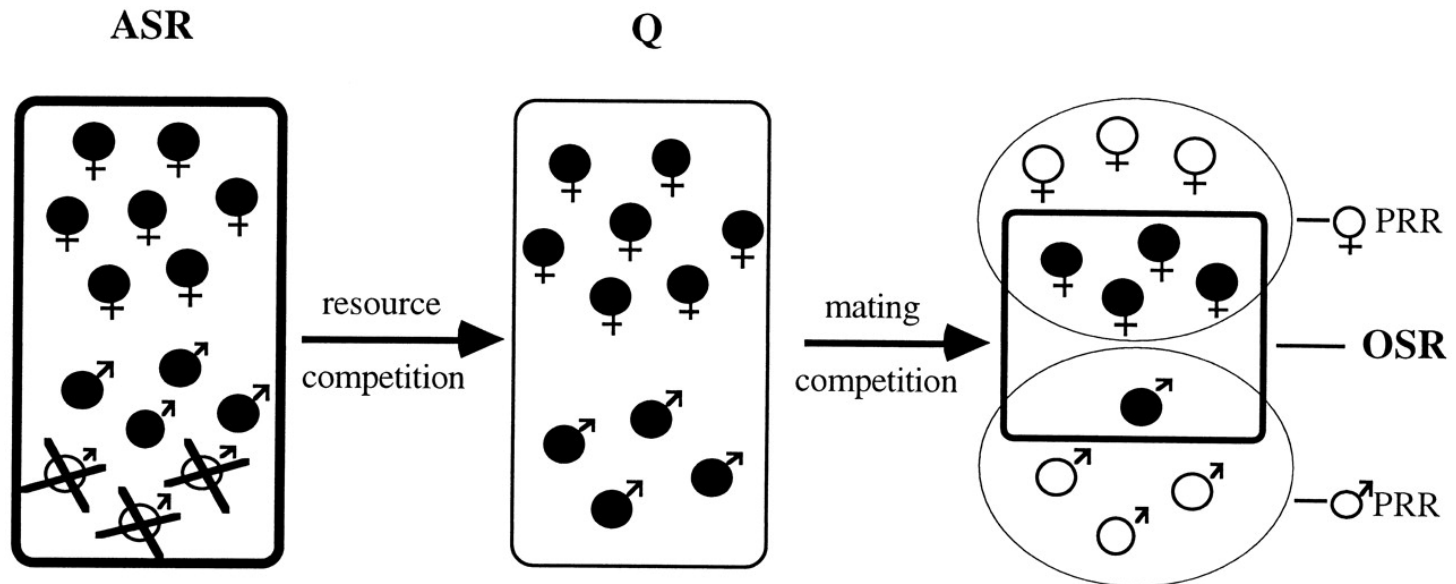
The relative number of members of each sex willing (or able) to mate at any given time (Kemp & Macedonia 2006)

# Changes in Personnel

(Clutton-Brock & Vincent 1991; Clutton-Brock & Parker 1992; Ahnesjö et al. 2001; Forsgren et al. 2004)

A change in emphasis from *all* individuals that are *potentially* receptive,

To one that includes only *certain receptive* individuals at a *particular* time and in a *particular* place



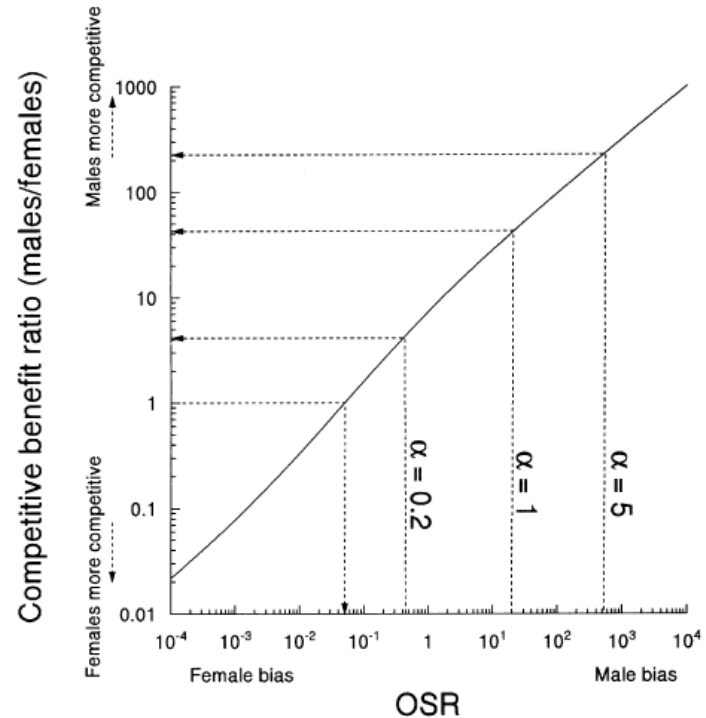
# Changes In Emphasis

(Clutton-Brock & Vincent 1991; Clutton-Brock & Parker 1992;  
Ahnesjö et al. 2001; Kokko & Monahagn 2001)

**The focus on parental care in sexual selection theory has also influenced the changing view of what OSR is.**

*“The **cost** of a single breeding attempt, which in iteroparous organisms can be measured as **the probability of death as a consequence of the current breeding attempt**, has a strong, direct effect on choosiness and well as consistent relationships with both OSR and PRR.”*

(Kokko & Monahagn 2001)





# Evolutionary Interpretations

## Biases in OSR are presumed to have significant consequences

**Variance in mating success:** (Positive effect: Emlen 1976; Balshine-Earn 1996; Kvarnemo et al. 1995; Jann et al. 2000; Jones et al. 2001; Foellmer & Fairbairn 2005; Negative effect: Shuster et al. 2001; No effect: Cerchio et al. 2005;

**Reversal of sex roles:** (Emlen & Oring 1977; Smith 1984; Berglund et al. 1989; Forsgren et al. 2004; Andersson 2005; Simmons & Kvarnemo 2006)

**Avoidance of sperm competition** (Positive: Møller 1989; Møller & Briskie 1995; Hosken 1997; Bateman 1997; Pitnick & Karr 1996; Negative: Pen & Weissing 1999; Kemp & Macedonia 2007)

**Mate selection and choosiness:** (Rosenqvist 1993; Berglund 1994; Kokko & Monaghan 2001)

**Mate guarding/mating duration** (McLain 1981; Sillen-Tullberg 1981; Jormalainen 1998; Gao & Kang 2005)

**Family sex ratio adjustment** McLain & Marsh 1990; Lopez & Dominguez 2003; Warner & Shine 2007;

**Aggressive behavior** Grant et al. 2000; Grant & Foam 2002;

**Changes in oviposition rate** Spence & Smith 2005;

**Female body temperature:** (Alsop et al. 2006)

**Population declines:** (Stifetten & Dale 2006)

# Summary 1

The definition of OSR has *changed* somewhat since 1976.

Working definitions of OSR have attempted *to improve the fit* between theory and observation.

Evolutionary predictions from observed biases in OSR are now *common*.

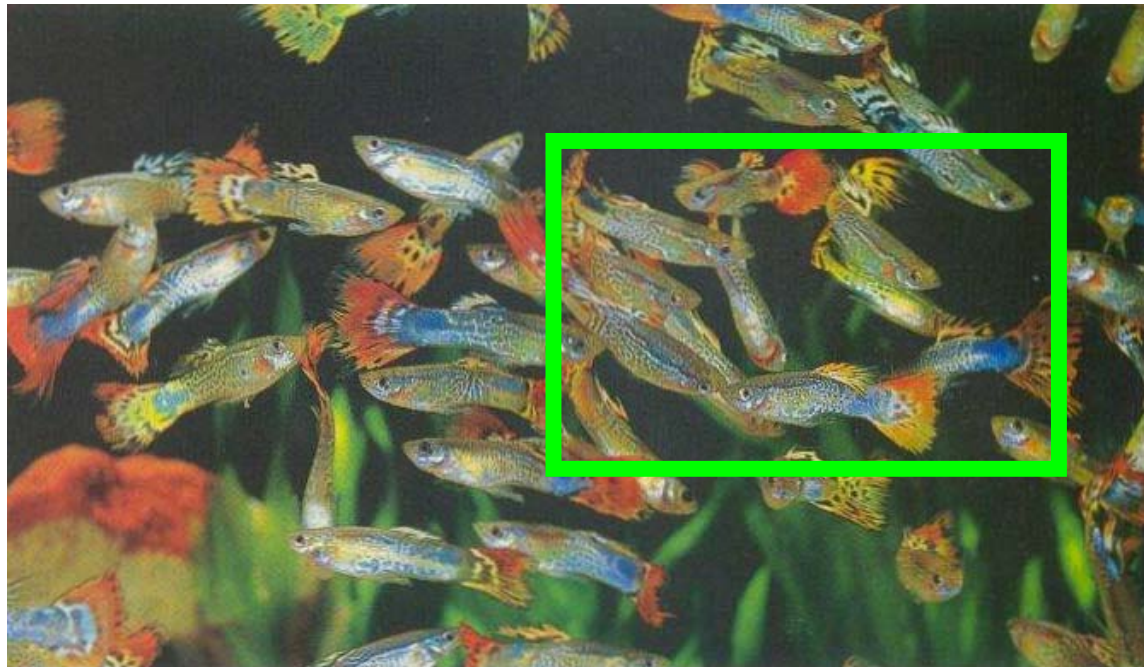
# Objectives

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3. Do better measures exist?

# Measuring OSR/PRR

(Clutton-Brock & Vincent 1991; Clutton-Brock & Parker 1992; Parker & Simmons 1996; Ahnesjö et al. 2001; Forsgren et al. 2004)

Considers the effect of *certain receptive* individuals at a *particular* time and in a *particular* place, on the **intensity of sexual selection**.



# Problems with Leaving Certain Individuals Out

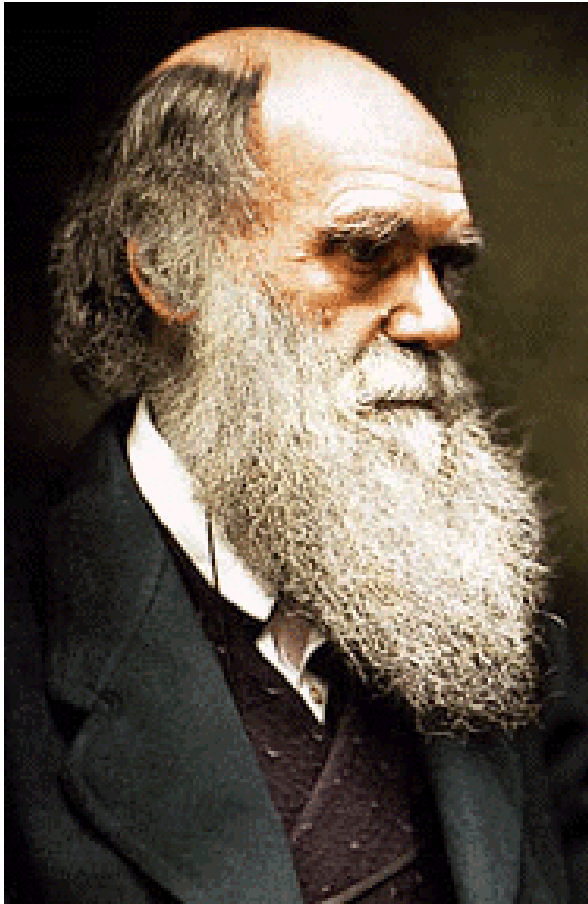
The justification for this is that only *certain* individuals reproduce at any time;

Including everyone *could bias* estimates of competition intensity.

Specifically, leaving individuals out **causes errors** in estimates of *actual selection*.



# The Cause of Sexual Selection

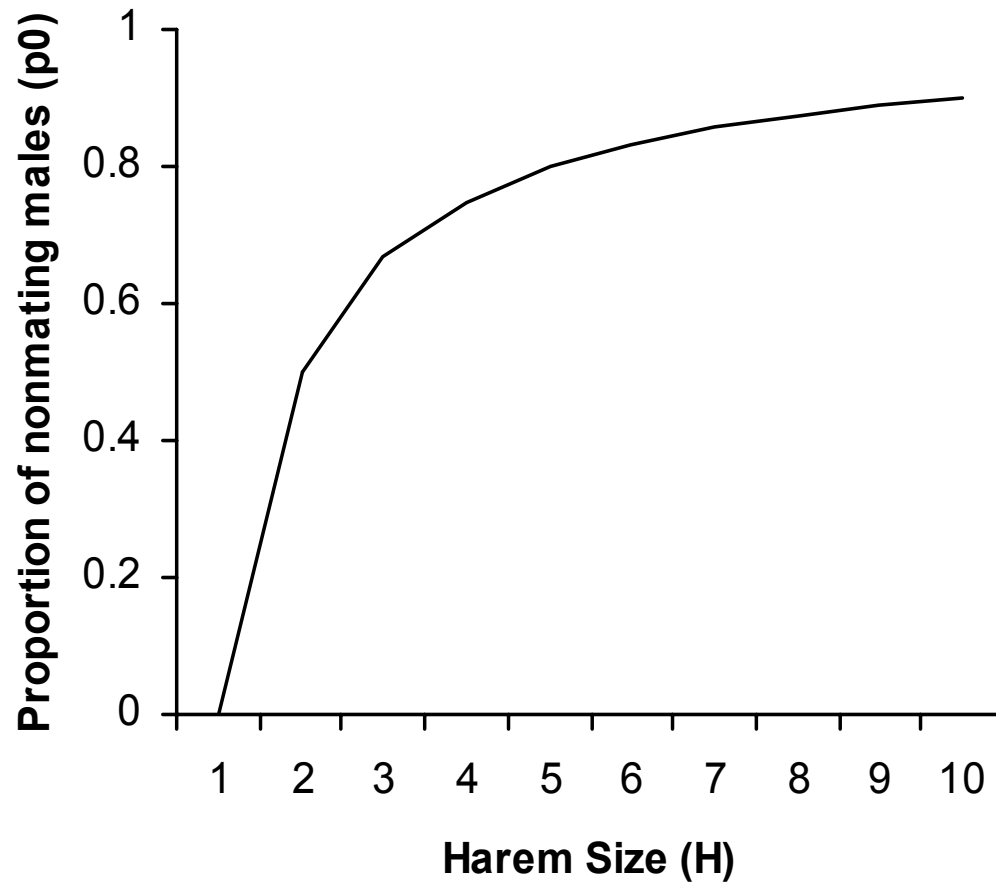


*“If each male  
secures two or more  
females, many  
males would not be  
able to pair”*

(Darwin 1871, p.  
266).

# For Harems, $p_0 = 1 - (1/H)$

Shuster & Wade 2003; Wade & Shuster 2004



# Sexual Selection is a Powerful Evolutionary Force Because:

For every male who sires young  
with  $k$  females, there must  
be  $k-1$  males who  
*fail to reproduce at all.*

Shuster & Wade 2003

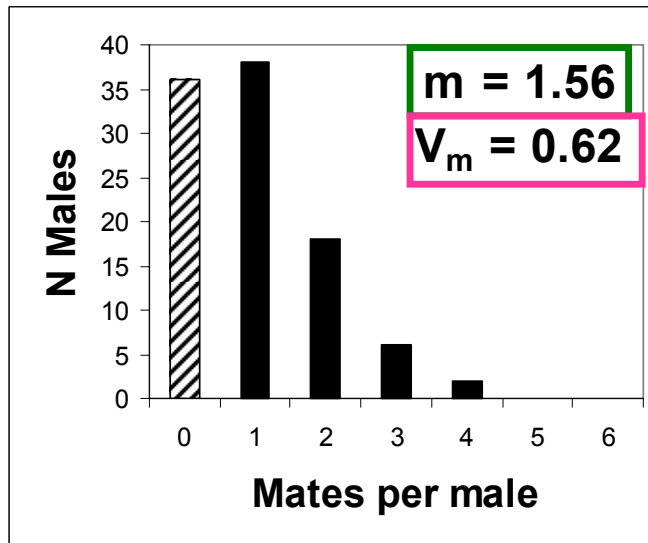
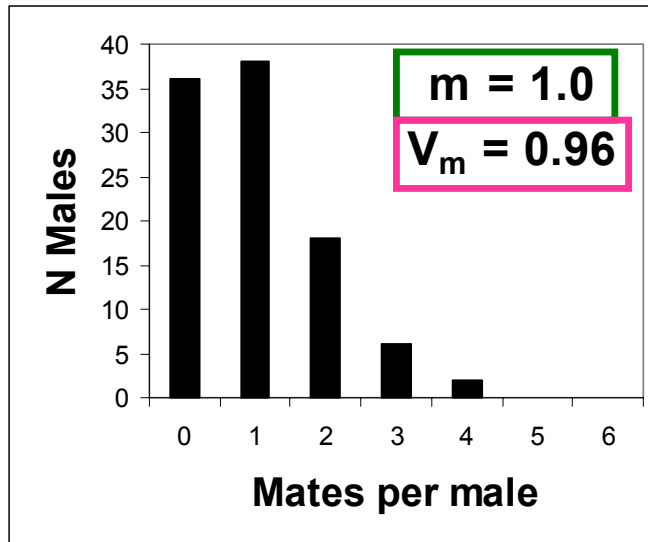


# When Losers are Ignored

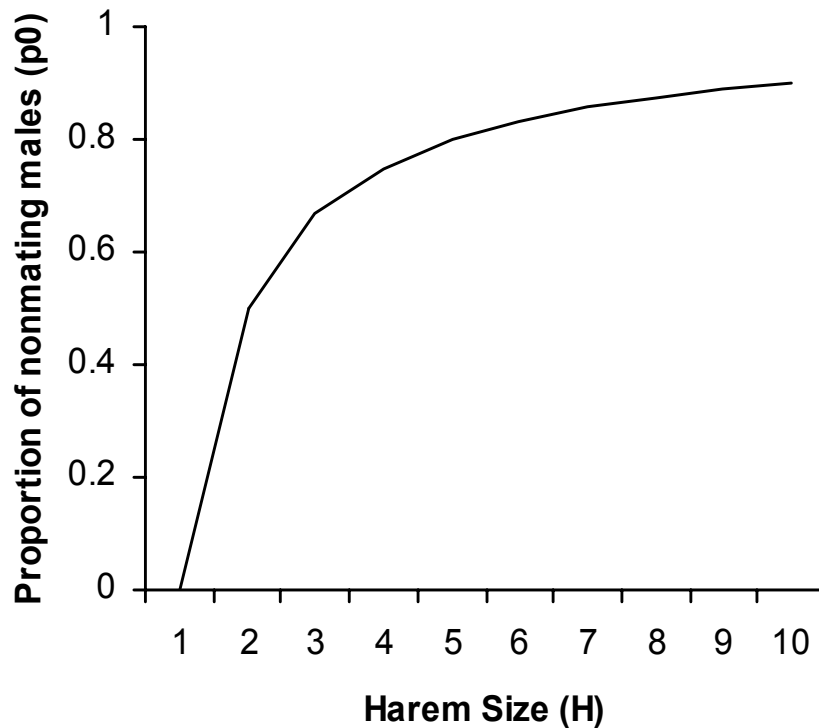
A significant fraction of the *among-group* component of fitness variance *goes unrecognized*.

This creates 2 kinds of errors:

1. The **average fitness** of the population is *overestimated*
2. The **variance in fitness** for the population is *underestimated*



# And, the Stronger Sexual Selection Becomes,



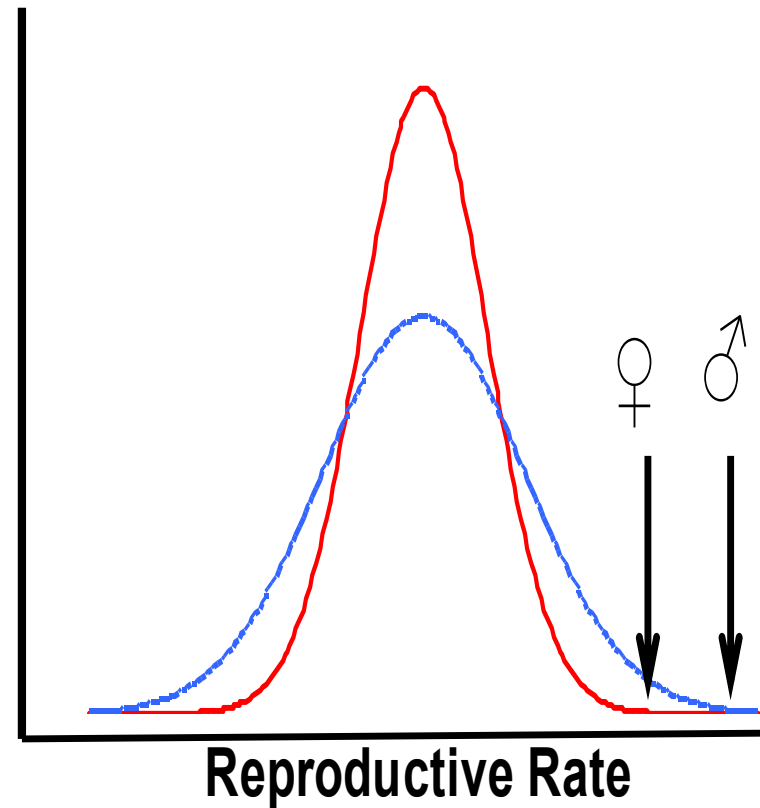
The *larger the possible error!*

Because as *fewer* males mate, *more* of the male population is *excluded* from mating altogether.

# A Similar Problem Exists for Potential Reproductive Rates

Only *a fraction* of the actual population is considered in most measurements – Those with the *largest potential values*

Under most circumstances, *few if any* individuals may achieve this rate.

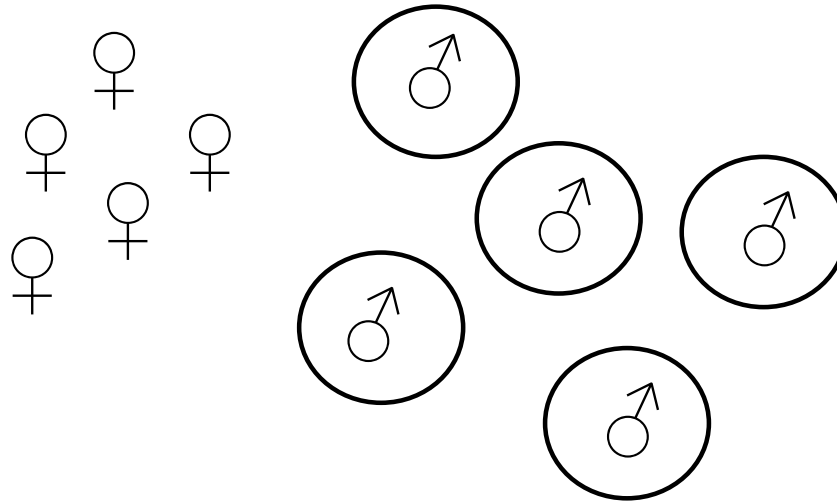


# Do OSR/PRR Really Measure What We Think They Measure?

**Does OSR reliably estimate the intensity of competition?**

Consider: Equal sex ratio (5 males: 5 females).

5 male territories, 5 females with variable receptivity



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Consider: Equal sex ratio (5 males: 5 females).

5 male territories, 5 females with variable receptivity

Patches w/males	Intervals w/ females->					$N_i$
	1	2	3	4	5	
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
$N_j$	0	0	0	0	0	0
					$\Sigma$	0

c.f. Shuster & Wade 2003

# Possible Measurements:

Patches w/ males	Intervals w/ females->					$N_i$
	1	2	3	4	5	
1	1	0	0	0	0	1
2	1	0	0	0	0	1
3	1	0	0	0	0	1
4	1	0	0	0	0	1
5	1	0	0	0	0	1
$N_j$	5	0	0	0	0	5
$K(t)$	5	5	5	5	5	5
$R(t)$	1.00	0.00	0.00	0.00	0.00	1.00
$R_o(t)$	1					1.00
$1/N_{females}(t)$	0.2					

$N_{.j} = N_{females}$  in each interval

$N_{i.} = N_{females}$  in each row

$K(t) = N_{males}$  in all territories

$R = N_{females}/N_{males}$

$R_o = N_{males}/N_{females} =$   
**OSR**

$R(t) = N_i/K(t) = R$  at each interval

$R_o(t) = K(t)/N_{i.} = R_o$  at each interval

$\Sigma R_o(t)$  = the sum of the individual instantaneous OSRs

$\Sigma R(t)$  = the sum of the individual instantaneous Rs

$1/N_{females}(t) = 1/N_{i.}$

# Scenario 1:

Intervals w/ females->

Patches w/ males	1	2	3	4	5	$N_i$
1	1	0	0	0	0	1
2	1	0	0	0	0	1
3	1	0	0	0	0	1
4	1	0	0	0	0	1
5	1	0	0	0	0	1
						$\Sigma$ 5
$N_j$	5	0	0	0	0	5
$K(t)$	5	5	5	5	5	
$R(t)$	1.00	0.00	0.00	0.00	0.00	1.00
$Ro(t)$	1					1.00
$1/N_{\text{females}}(t)$	0.2					

# Scenario 2:

		Intervals w/ females->					$N_i$
Patches w/ males		1	2	3	4	5	
1		1	1	1	1	1	5
2		0	0	0	0	0	0
3		0	0	0	0	0	0
4		0	0	0	0	0	0
5		0	0	0	0	0	0
		$\Sigma$					5
$N_j$		1	1	1	1	1	5
$K(t)$		5	5	5	5	5	
$R(t)$		0.20	0.20	0.20	0.20	0.20	1.00
$Ro(t)$		5	5	5	5	5	25.00
$1/N_{\text{females}}(t)$		1	1	1	1	1	



# Scenario 3:

Patches w/ males	Intervals w/ females->					$N_{i.}$
	1	2	3	4	5	
1	1	0	0	0	0	1
2	0	1	0	0	0	1
3	0	0	1	0	0	1
4	0	0	0	1	0	1
5	0	0	0	0	1	1
					$\Sigma$	5
$N_{.j}$	1	1	1	1	1	5
$K(t)$	5	5	5	5	5	
$R(t)$	0.20	0.20	0.20	0.20	0.20	1.00
$Ro(t)$	5	5	5	5	5	25.00
$1/N_{\text{females}}(t)$	1	1	1	1	1	

## Scenario 2:

		Intervals w/ females->					$N_i$
Patches w/ males		1	2	3	4	5	
1	1	1	1	1	1	1	5
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
$N_j$	1	1	1	1	1	1	$\Sigma$ 5
$K(t)$	5	5	5	5	5	5	5
$R(t)$	0.20	0.20	0.20	0.20	0.20	0.20	1.00
$R_o(t)$	5	5	5	5	5	5	25.00
$1/N_{\text{females}}(t)$	1	1	1	1	1	1	

## Scenario 3:

		Intervals w/ females->					$N_i$
Patches w/ males		1	2	3	4	5	
1	1	0	0	0	0	0	1
2	0	1	0	0	0	0	1
3	0	0	1	0	0	0	1
4	0	0	0	1	0	0	1
5	0	0	0	0	1	1	1
$N_j$	1	1	1	1	1	1	$\Sigma$ 5
$K(t)$	5	5	5	5	5	5	5
$R(t)$	0.20	0.20	0.20	0.20	0.20	0.20	1.00
$R_o(t)$	5	5	5	5	5	5	25.00
$1/N_{\text{females}}(t)$	1	1	1	1	1	1	

# But there is a Problem...

Scenarios 2 and 3 are *clearly distinct* in the number of males that mate.

Yet their instantaneous OSRs [ $R_o(t)$ ] *are identical!*

Also, while  $\Sigma R(t) = R = R_o = 1$ ,

$$\Sigma R_o(t) = 25.$$

Each  $R_o(t)$  *overestimates* the overall effect of competition among males.

## Scenario 2:

Patches w/ males	Intervals w/ females->					$N_i$
	1	2	3	4	5	
1	1	1	1	1	1	5
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
	$\Sigma$					5
$N_j$	1	1	1	1	1	5
$K(t)$	5	5	5	5	5	
$R(t)$	0.20	0.20	0.20	0.20	0.20	1.00
$Ro(t)$	5	5	5	5	5	25.00
$1/N_{females}(t)$	1	1	1	1	1	

## Scenario 3:

Patches w/ males	Intervals w/ females->					$N_i$
	1	2	3	4	5	
1	1	0	0	0	0	1
2	0	1	0	0	0	1
3	0	0	1	0	0	1
4	0	0	0	1	0	1
5	0	0	0	0	1	1
	$\Sigma$					5
$N_j$	1	1	1	1	1	5
$K(t)$	5	5	5	5	5	
$R(t)$	0.20	0.20	0.20	0.20	0.20	1.00
$Ro(t)$	5	5	5	5	5	25.00
$1/N_{females}(t)$	1	1	1	1	1	

## Why?

Because  $\Sigma R_o(t) = 25$   
equals

$$(N_{males}/H_{females})T$$

Where  $T$  = the number of  
intervals containing  
females (=5),

and  $1/H_{females}$  is the  
**harmonic mean** number  
of females per interval,  
 $= (1/T)(\Sigma 1/N_{females}) = 1$

# The Harmonic Mean

$$1/\mathbf{H}_{females} = (1/T)(\Sigma 1/\mathbf{N}_{females})$$

Allows small values to contribute *disproportionately* to the value of  $1/\mathbf{H}_{females}$ .

Because  $\mathbf{R}_O(t) = \mathbf{N}_{males}/\mathbf{N}_{females}$ ,

*apparently high* levels of male competition will *disproportionately contribute* to the overall value of  $\Sigma \mathbf{R}_O(t)$ .

## Summary 2

Current methods for measuring OSR/PRR tend to *cause errors* in estimates of the selection intensity.

Estimates can be too high, too low, or simply not representative of how selection *actually operates*.

Presumed evolutionary consequences of biased OSRs are likely also *erroneous*.

# Objectives

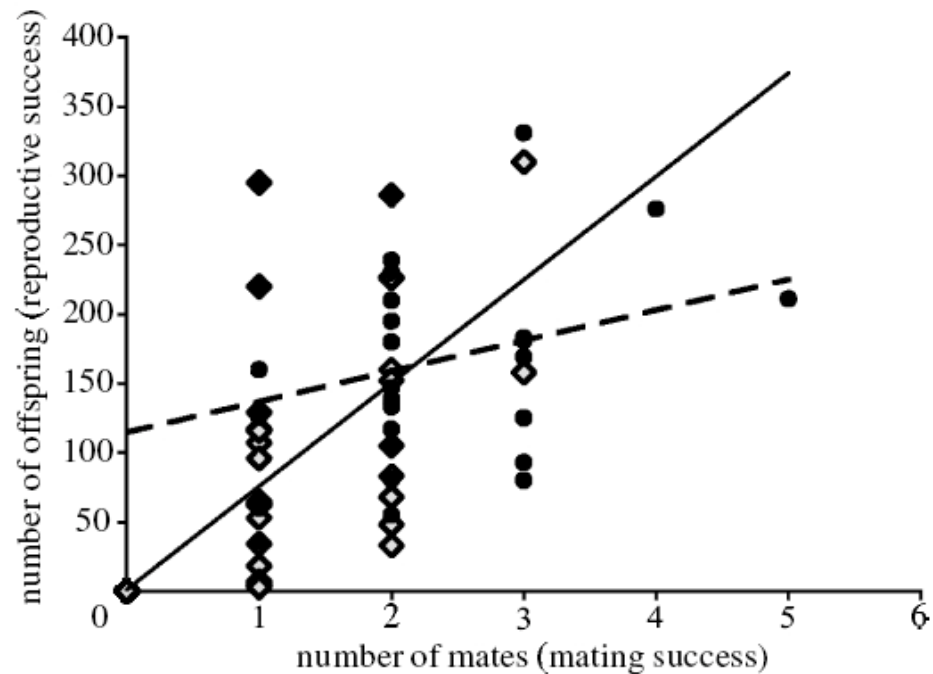
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# The Best Approach

## *Measure Selection Directly*

**If traits under selection are known,**

Measure the standardized covariance between phenotype and fitness – slope of this line is  $\beta$ .



# The Opportunity for Selection

(Crow 1958, 1962; Wade 1979)

$$I = V_w / W^2 = V_w$$

Compares the fitness of breeding parents *relative* to the population before selection.

The **variance in relative fitness**,  $V_w$ , provides an empirical estimate for selection's strength.

$$I_{\text{♂}} = V_{o_{\text{♂}}} / (O_{\text{♂}})^2 \quad I_{\text{♀}} = V_{o_{\text{♀}}} / (O_{\text{♀}})^2$$



# The Sex Difference in the Opportunity for Selection, $I_{\text{mates}}$

Wade 1979; Arnold & Wade 1983; Shuster & Wade 2003

$$\{ I_{\text{♂}} - I_{\text{♀}} \} = I_{\text{mates}}$$

The relative magnitudes of  $I_{\text{♂}}$  and  $I_{\text{♀}}$  determine whether sexual selection modifies males or females

**What If You Can't Assign Parentage Easily?**

Patches w/males	Intervals w/ females->					$N_i$	$N_i/T$	$V(k)$	$I_{\text{mates}}(k)$
	1	2	3	4	5				
1	1	1	1	1	1	5	1	0.00	0
2	0	0	0	0	0	0	0	0.00	
3	0	0	0	0	0	0	0	0.00	
4	0	0	0	0	0	0	0	0.00	
5	0	0	0	0	0	0	0	0.00	
$\Sigma$						5			

$N_j$	1	1	1	1	1
$K(t)$	5	5	5	5	5
$(R_t)$	0.20	0.20	0.20	0.20	0.20
$Ro(t)$	5	5	5	5	5
$V_{ti}$	0.16	0.16	0.16	0.16	0.16
$I_{\text{mates}}(t)$	4	4	4	4	4
$(R(t)/R)^2$	0.04	0.04	0.04	0.04	0.04
$*I_{\text{mates}}(t)$	0.16	0.16	0.16	0.16	0.16

5 1 **Shuster & Wade 2003**  
pp. 94-100

$$V_{Nij} = \bar{V}_{\text{mates}(t)} + V_{(R[t])}$$

$$V_{Nij} = \bar{V}_{t(k)} + (V_{\text{mates}})/(T)^2$$

$$I_{\text{mates}} = (T/R)^2 \{ V_{(R[t])} + (\bar{V}_{\text{mates}(t)} - \bar{V}_{t(k)}) \}$$

Patches w/males	Intervals w/ females->					$N_i$	$N_i/T$	$V(k)$	$I_{mates}(k)$
	1	2	3	4	5				
1	1	1	1	1	1	5	1	0.00	0
2	0	0	0	0	0	0	0	0.00	
3	0	0	0	0	0	0	0	0.00	
4	0	0	0	0	0	0	0	0.00	
5	0	0	0	0	0	0	0	0.00	
	$\Sigma$					5			

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$I_{mates}(t)$	4	4	4	4	4	
$(R(t)/R)^2$	0.04	0.04	0.04	0.04	0.04	
$*I_{mates}(t)$	0.16	0.16	0.16	0.16	0.16	

**Shuster & Wade 2003**  
pp. 94-100

$$N_{females}/N_{males} = R$$

$$I_{mates} = V_{mates}/R^2$$

$$I_{mates} = (T/R)^2 \{ V_{(R[t])} + (\bar{V}_{mates(t)} - \bar{V}_{t(k)}) \}$$

Patches w/males	Intervals w/ females->					$N_i$	$N_i/T$	$V(k)$	$I_{mates}(k)$
	1	2	3	4	5				
1	1	1	1	1	1	5	1	0.00	0
2	0	0	0	0	0	0	0	0.00	
3	0	0	0	0	0	0	0	0.00	
4	0	0	0	0	0	0	0	0.00	
5	0	0	0	0	0	0	0	0.00	
	$\Sigma$					5			

$N_j$	1	1	1	1	1	5
$K(t)$	5	5	5	5	5	
$(R_t)$	0.20	0.20	0.20	0.20	0.20	1.00
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$V_{ti}$	0.16	0.16	0.16	0.16	0.16	
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$(R(t)/R)^2$	0.04	0.04	0.04	0.04	0.04	
$*I_{mates}(t)$	0.16	0.16	0.16	0.16	0.16	

**Shuster & Wade 2003**  
pp. 94-100

$$N_{females}/N_{males} = R$$

$$I_{mates} = V_{mates}/R^2$$

$$I_{mates} = I_{sex\ ratio} + (*I_{mates}(t) - *I_{mates}(k))$$

# What Does It Mean?

$$I_{\text{mates}} = I_{\text{sex ratio}} + (*I_{\text{mates}(t)} - *I_{\text{mates}(k)})$$

**The total opportunity for sexual selection**

The opportunity for sexual selection caused by temporal variation in the *sex ratio* (a better ‘OSR’)

The opportunity for sexual selection caused by *temporal variation* in the availability of females

*minus*

The opportunity for sexual selection caused by *spatial variation* in the availability of females

Intervals w/ females->

1 2 3 4 5

1	1	0	0	0	0
2	1	0	0	0	0
3	1	0	0	0	0
4	1	0	0	0	0
5	1	0	0	0	0

$\Sigma R(t)$

$R_o$

$\Sigma R_o(t)$

$\bar{V}_{mates}(t)$

$V(R[t])$

$\bar{V}_t(k)$

$V_{mates}/(T)^2$

$V_{females}(t)$

$V_{mates}$

$I_{sexratio}$

$*I_{mates}(t)$

$*I_{mates}(k)$

$I_{mates}$

1	1	1	1	1	1
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0

1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1

Scenario  
1

Scenario  
2

Scenario  
3

1.00

1.00

1.00

1.00

1.00

1.00

1.00

25.00

25.00

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# Mating Systems and Sexual Selection

(Emlen & Oring 1977)



- Male reproduction is limited by the *spatial distribution* of resources and by the *temporal distribution* of sexually receptive females.

- By partitioning the total variance in  $N_{females}$ , the *intensity of sexual selection*

can be directly measured.

$$I_{mates} = I_{sex\ ratio} + (*I_{mates(t)} - *I_{mates(k)})$$

# Summary 3

*Better* parameters than OSR, for estimating temporal variation in the intensity of sexual selection, *do exist*.

They require more *precise* estimates of who mates and who does not.

Evolutionary predictions from observed biases in OSR/PRR *must be reconsidered*.



