BIO 682
Nonparametric Statistics
Spring 2010

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http://www4.nau.edu/shustercourses/BIO682/index.htm

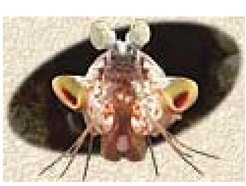
Lecture 6

2x2 Tests: *G*-test

1. Set up table in the same way as before
2. This time suppose we have two sets of stomatopods
   a. One set is males paired to females
   b. The other set is unpaired males
   c. Pairs allowed to fight against larger males.

2x2 Tests: *G*-test

	Rwins	Iwins	
Paired	18	2	20
Control	12	23	35
	30	25	55



For *G*-test, it is not necessary to calculate expected values, just need to use:

```
G = 2[(Σn_ij ln n_ij) - (Σm_rc ln m_rc) + N ln N]
  = 2[(18 ln 18 + 2 ln 2 + 12 ln 12 + 23 ln 23) -
      (20 ln 20 + 35 ln 35 + 30 ln 30 + 25 ln 25) +
        55 ln 55]
  = 17.78, df =(r-1)(c-1) = 1
```

	Rwins	Iwins	
Paired	18	2	20
Control	12	23	35
	30	25	55

2x2 Tests: *G*-test

1. And since N < 200, need to apply Williams' Correction:

a. $G_{adj} = G/[1 + (1/2n)]$

$= 17.78/1.0025 = 17.74$

Not much change with large sample size

$X^2_{[.001]} = 10.83$, P<0.001.

b. Conclusion: paired males are more successful against larger male intruders than unpaired males.

Note Marginal Totals

1. First one: both marginal totals can vary.
2. Second one: one set of the marginal totals is determined by the experimenter.
3. These are situations appropriate for *G*-tests.
4. What if marginal totals are ***fixed***?

		Plant type		
		Hy	WT	
Insect	-	4	14	18
A	+	32	50	82
		36	64	100

	Rwins	Iwins	
Paired	18	2	20
Control	12	23	35
	30	25	55

Fisher's Exact Test

1. Another form of 2x2 table

2. This time consider electrophoretic data at a single locus:

a. *Pgm* electromorphs 1 and 2.

3. Also, male phenotype:α-male and γ-male (controlled by alleles at single locus).

c. Genetic cross between

α||γ, 2||1 γ-male X   α||α, 1||1 female

Fisher's Exact Test

	PGM 1	PGM 2	
a-males	8	2	10
γ-males	2	8	10
	10	10	20

Fisher's Exact Test

1. Here, we calculate the exact probability of this distribution for N = 20.

a. Useful for small sample sizes.

b. Situations when marginal totals are ***fixed***

c. Not exactly true for this case.

d. Except that Mendelian genetics dictates 50:50 ratios.

e. It is also possible to have these frequencies vary.

Fisher's Exact Test

Calculation of *p*:

$$= \frac{(A+B)!\,(C+D)!\,(A+C)!\,(B+D)!}{N!\,A!\,B!\,C!\,D!}$$

	PGM 1	PGM 2	
a-males	8	2	10
Y-males	2	8	10
	10	10	20

Fisher's Exact Test

$$= \frac{10!\,10!\,10!\,10!}{20!\,8!\,2!\,2!\,8!} = .0196$$

$P_1 = .00054; \quad P_2 = .00000541 \quad \sum P = .0199$

	PGM 1	PGM 2	
a-males	8	2	10
Y-males	2	8	10
	10	10	20

Calculation of *p*:

$$= \frac{(A+B)!\,(C+D)!\,(A+C)!\,(B+D)!}{N!A!B!C!D!}$$


TABLE I
Probabilities for Fourfold tables, Fisher exact test, $N \leq 15$*

These tables are extracted from more extensive tables prepared by Donald Goyette and M. Ray Mickey, Health Science Computing Facility, UCLA.

Models I-III

1. A scheme for determining which 2x2 test to use.
2. Similar to models for ANOVA in determining whether factors are fixed or random.

Model I

a. Total is fixed, but marginal totals *not fixed*.

b. Both variables can vary as a function of the treatment.

c. Use a G-test

		Plant type		
		Hy	WT	
Insect	-	4	14	18
A	+	32	50	82
		36	64	100

Model II

a. One criterion fixed (female present/female absent), determined by experimenter.

b. Behavior of males is free to vary

c. Use a G-test

	Rwins	Iwins	
Paired	18	2	20
Control	12	23	35
	30	25	55

Model III

a. Both criteria are fixed, two factors, two specific treatments

b. use Fisher's exact test.

	PGM 1	PGM 2	
a-males	8	2	10
γ-males	2	8	10
	10	10	20

k-Sample Tests

1. Tests in which population is sampled *multiple times*.

2. Good example: Cochran's Q test.

a. A test for nominal scale data that tests for *changes over time*

b. Similar to McNemar's test, except that duration is not limited to two samples.

Tests of 2 Related Samples

1. McNemar's test

a. Test is used to determine if sample has changed in character over time.

b. Note non-independence *implicit* in this test.

2. Examples:

a. Do experimental animals adapt to laboratory conditions?

b. Does treatment influence behavior of same group of individuals?

McNemar's Test: Method

1. Involves examining the frequencies at time t_1 and t_2 and placing them into a *contingency table.*

2. A frequently used technique: 2x2 table:

t_2/t_1	-	+
-	**A**	**B**
+	**C**	**D**

McNemar's Test: Method

t_2/t_1	-	+
-	**A**	**B**
+	**C**	**D**

1. The number of individuals that change is B + C.

a. Thus if changes were at random, *expected frequencies* for these cells is (B + C)/2.

McNemar's Test: Method

2. Test involves comparing observed and expected values for B+C.

a. It is possible to use X^2 test with df = 1

b. Also possible to use *G*-test with df = 1

1. Same precautions as before for either test:

a. Large samples - use *G*-test

b. Small samples - use *G*-test with Williams' Correction.

c. Very small samples - use exact probability.

McNemar's Test: Method

Faculty Member:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Before Backman:	+	-	-	-	-	+	-	-	-	+	+	-	-	-	-	+	+	-	-	-
After Backman:	-	+	+	+	-	+	+	-	+	+	+	-	-	+	+	-	+	+	-	-

		Before Backman -	Before Backman +	
After	-	6	2	8
Backman	+	8	4	12
		14	6	20

McNemar's Test: Method

		Before Backman -	Before Backman +	
After	-	6	2	8
Backman	+	8	4	12
		14	6	20

$G = 2\{B \ln [B/(C+B/2)] + C \ln [C(B+C/2)]\}$

$= 2\{2 \ln [2/5] + 8 \ln [8/5]\} = 3.85$

$X^2_{[.05]} = 3.84$, $P<0.05$

McNemar's Test: Method

		Before Backman -	Before Backman +	
After	-	6	2	8
Backman	+	8	4	12
		14	6	20

$G = 3.85$

But with William's correction

$q = 1 + [1/2n] = 1 + [1/40]$

$= 1.025$

$G_{adj} = 3.85/1.025 = 3.76$ ns

k-Sample Tests

1. Tests in which population is sampled *multiple times*.

2. Good example: Cochran's Q test.

a. A test for nominal scale data that tests for *changes over time*

b. Similar to McNemar’s test, except that duration is not limited to two samples.

Cochran’s Q Test

1. Faculty responses to New Plan at various times in over the last few months.

a. Possible to examine the effect of time on subjects.

b. Useful to have a control in most cases (not always possible).

Cochran’s Q Test: Method

1. examine matrix with:

a. a = # of columns (sampling events)

b. b = # of rows (subjects)

c. Y = score for each individual at time a_i (0 or 1)

Cochran's Q Test: Method

b_i	a_1	a_2	a_3	Σ	
1	0	1	0	1	1
2	0	0	0	0	0
3	1	1	0	2	4
4	1	0	0	1	1
5	1	1	0	2	4
6	1	0	0	1	1
7	0	0	0	0	0
8	1	1	1	3	9
9	1	1	0	2	4
10	1	0	0	1	1
Σ	7	5	1	13	25
	49	25	1	75	

Calculate:

1. $\sum\sum^{ab} Y_i = 13 = A$
2. $\sum^{b}(\sum^{a} Y_i)^2 = 25 = B$
3. $\sum^{a}(\sum^{b} Y_i)^2 = 75 = C$

Cochran's Q Test: Method

$$Q = \frac{(a-1)\,[a\,(C) - (A)^2]}{a\,(A) - B}$$

$$= \frac{(3-1)\,[3(75) - 169]}{3(13) - 25}$$

$$= 8.$$

with df = $(a-1) = 2$, ($X^2 = 5.99$), $P < .05$

People change.

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---

b_i	a_1	a_2	a_3	Σ	
1	0	1	0	1	1
2	0	0	0	0	0
3	1	1	0	2	4
4	1	0	0	1	1
5	1	1	0	2	4
6	1	0	0	1	1
7	0	0	0	0	0
8	1	1	1	3	9
9	1	1	0	2	4
10	1	0	0	1	1
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= 8.

with df = (a-1) = 2, (X^2 = 5.99), P < .05

People change.

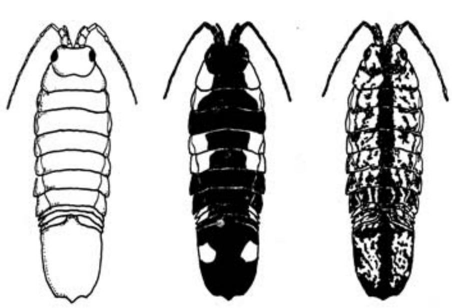
RxC Tests

1. Like a contingency table but with *more than* 2 rows or columns.
2. The classic method: RxC X^2 test.
   a. Method:
   1. Marginal values calculated as with 2x2 test.
   2. Add up all X^2 values for cells.
3. Has same problems with being cumbersome as 2x2 X^2 test.

RxC *G*-tests

1. Has same advantages as before, same rules:
   a. for $a > 5$ and $f_{i\text{-hat}} > 3$; *G* is *better* than X^2
   b. Use an exact test when:
      a. $a > 5$ and $f_{i\text{-hat}} < 3$
      b. $a < 5$ and $f_{i\text{-hat}} < 5$
2. Commonly used test to examine independence of multiple classes.

Example: *Idotea baltica*



uniformis *albufusca* *maculata*

Example: *Idotea baltica*

	M	J	J	A	
uniformis	254	185	93	55	587
albafusca	185	144	123	190	642
maculata	66	98	200	305	669
	505	427	416	550	1898
obs. *f* mac.	.13	.23	.48	.55	avg = .35

Example: *Idotea baltica*

1. RxC test allows you to test the hypothesis that the observed frequencies *don't change.*

2. Same method as 2x2: [(Σ*G*-cells)-(Σ*G*-rows)-(Σ*G*-columns)+(*G*-N)]

a. with df = (*r*-1)(*c*-1) = 6.

3. Williams' correction is used for sample sizes <200.

a. Is a lot more complicated than before (see p. 745).

Williams' Correction: RxC

$$q = \frac{\left(n\sum^{b} 1/\sum^{a} f - 1\right)\left(n\sum^{b} 1/\sum^{a} f - 1\right)}{6n(a-1)(b-1)}$$

But the shorter version provides a lower boundary (a conservative substitute),

$$q = 1 + [(a+1)(b+1)]/6n$$