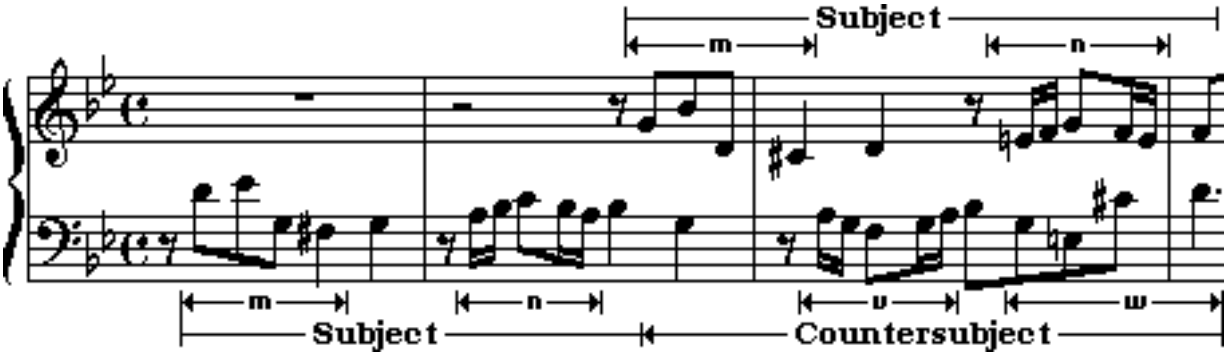
Fugue No. 16

G minor
Well-Tempered Clavier Book I
Johann Sebastian Bach

To read this essay in its hypermedia format, go to the Shockwave movie at http://www2.nau.edu/tas3/wtc/i16.html.



Subject: Fugue No. 16, *Well-Tempered Clavier*, Book I

This fugue is an uncommon treasure. In its clever generation of the countersubject from the subject, and nested layers of double counterpoint, this fugue:

- is like a fractal
- mirrors the subject
- varies both motive and texture
- connects time
- gives voice to the imagination

Is Like a Fractal

You've probably heard of these weird and wonderful things; maybe you've even listened to fractal music or have been dazzled by fractal images like Bruce Dawson's to the right. While the beauty of a fractal is easy to see, it is difficult to explain. This is because, while the fractal's blueprint is simple, its product is exceedingly complex. What you see is the representation of an abstraction.

A fractal is the representation of a simple formula: (i x i)+c equals the new value for i. Variables that are repeatable without producing an absolute value of 2

will generate fractals. Insofar as the generated patterns are never exactly repeated, they are chaotic. Yet amazingly there is order within the chaos.

Observing a fractal's order is like tunneling into the reflection of your reflection in facing mirrors. Each reflection shows, alternately, your face and the back of your head. Each reflection is successively smaller and at a slightly increased angle. Now study the fractal images to the right. With a magnifying glass you could zoom in to discover that each enlargement resembles the whole and that the whole resembles its parts. That is what a fugue is like.

Mirrors the Subject

Like most fugues, this subject has two parts: head and tail. This fugue also contains a countersubject (listen to the low voice). Normally, a fugue's countersubject is new material. But in this fugue the counter is generated from the subject. The subject's head motive is mirrored by the countersubject's tail, and the subject's tail motive is mirrored by the countersubject's head.

I have labeled each of these elements in the score. Figure m is in the subject's head and figure n in its tail. The countersubject's figure u is the inversion of n, and the countersubject's figure w (low voice) is the inversion of m. The underlying motivic complex is shown on the score and in the following diagram:

Subject = m + n
Countersubject = u + w
(where n/u and m/w are the melodic inversions of each other)

Remember this: (1) the countersubject is generated from the subject by (2) reversing the order of the subject's head and tail and (3) moving their intervals in the opposite direction (melodic inversion).

Varies Both Motive and Texture

The fact that its countersubject is generated from its subject is enough to make the g-minor fugue like a fractal. But it also contains self-similar structures on two more scales. The creative mechanism is *double counterpoint,* a technique whereby simultaneously sounding melodies are repeated with an exchange of registers. What had been in the low voice recurs in the high and vice versa.

The double counterpoint of this fugue exists at two levels: motivic and textural. At the first level, motives m/n & u/w comprise the earlier highlighted complex. To appreciate the genius of this fugue we should understand exactly how this complex transforms the motive. It is not only by inversion and retrograde order, but also by an exchange of registers (i.e. double counterpoint). Every time we see a migration of m/n & u/w, double counterpoint has transpired at the level of the motive. You may think of inversion and retrogradation as Bach's formula for *motivic variation.* In this fugue the motivic variation is accompanied by small-scale double counterpoint.

On a larger scale, double counterpoint yields a substantive *textural variation.* The complex of subject (gray) and countersubject (cream) is repeated twelve times with no two being exactly alike. To insure variation Bach has employed double counterpoint at a level higher than the motive.

In all but two adjacent complexes the subject and countersubject have exchanged registers. Every time we see migration of gray and cream rectangles, double counterpoint has transpired at the textural level. But, in the special case of this fugue, every textural exchange *also* generates a motivic exchange. Like the old spiritual where "Ezekiel saw the wheel within a wheel way up in the middle of the air," the fugue's double counterpoint contains others that have been nested within. That is how this fugue is like a fractal.

This is not to imply that the fugue is a fractal. Unlike a fractal the fugue is neither generated from a mathematical equation nor is it chaotic. But insofar as they both involve a process that produces self-similar structures at various levels, fugues and fractals are alike. Just as a fractal's shape is predetermined by its mathematical formula, the fugal process is predetermined by the contrapuntal possibilities inherent in its subject. The fugue's shape is a product of its kernel idea.

Connects Time

Another way in which the fugue is like a fractal is that both require the intersection of time. Fractal formulae are infinitely repeatable. Relentless (a fractal composition by Phil Jackson) could theoretically continue forever without exact repetition. Similarly one could tunnel into Bruce Dawson's fractal images and never see exactly the same thing in successively higher magnifications.

There is an interesting historical footnote to this fugue. In 1899 Seiffert observed that its subject and formal proportions were similar to the E-flat fugue from Johann Fischer's *Ariadne Musica* (1702). In Bach's day (as even today) an ingenious reworking of another composer's idea was not considered to be plagiarism but a compliment.

That Bach's work improves upon Fischer's does not devalue the contribution of the latter. Without Fischer, this fugue may not have come into existence. His work is relevant to this study in yet a more important way. You see, Fischer's *Ariadne Musica* was a cycle of twenty preludes and fugues in related keys. So it is likely that Fischer inspired not just this fugue but the whole of the *Well-Tempered Clavier!*

It is important to consider how Bach's work was rooted in a historical context. Like the fractal, his life and work provide a pattern intersecting with other patterns in time. As Bach varied upon Fischer, Brahms would later vary upon Bach, and Schoenberg upon Brahms. Making these types of connections is one of the most important ways of deriving meaning, not only in works of art but also about the nature and purpose of everything that exists.

Gives Voice to the Imagination

We have observed that the fractal is a representation of a unique class of mathematical formulae. These formulae are unique because they involve an

imaginary number--the square root of negative one. Big problem! There is no known number that when multiplied by itself will produce a negative.

Not to be dissuaded, mathematicians reasoned that if such a number did exist it should behave itself in an equation. Not knowing what the number was, they gave it a symbol nonetheless. The symbol was i (for imagination). To their surprise they found that i worked; the equations in which i behaved were those that produced fractals.

Bach's fugues, too, are expressions of a bountiful imagination. The process of analyzing them is an attempt to discover the i that produced them. In most cases the i can be theorized to be a particle of sound that, when combined with another particle or a contrapuntal derivative of itself, produces a motivic complex having the ability to replicate itself with slight variation.

A fascinating attribute of the fractal is that its various formulae (Mandelbrot set, Julia set etc.) can produce images resembling a wide variety of things that exist: clouds, forests, sand-dunes, ferns, pine trees, seashells, galaxies, nebulae, snowflakes, crystals, zebra stripes, gemstones, and dividing cells. They can even be used to forecast the weather, traffic patterns, and chemical reactions. That a fugue is also like a fractal, should heighten our appreciation of the beauty, and the mystery, of both.