Recall in a topological space a point $z$ is a cluster point of $A$ if for all open neighborhood $N$ of $z$ we have $(N \setminus \{z\}) \cap A \neq \emptyset$.

1. Let $X = \{a, b, c\}$ and $\mathcal{T} = \{\emptyset, \{a\}, \{b, c\}, X\}$ a topology on $X$. Find the following:
   a. all dense subsets of $X$
   b. $\{b\}$
   c. $\{a, b\}$
   d. $\partial\{b\}$
   e. all nowhere dense subsets of $X$.
   f. all isolated points of $X$.
   g. $\{c\}$

   Solution: a. $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$.
   b. $\{b, c\}$
   c. $\{a\}$
   d. $\partial\{b\} = \overline{\{b\}} \cap \overline{\{b\}}^C = \{b, c\} \cap \overline{\{a, c\}} = \{b, c\} \cap X = \{b, c\}$
   e. $\emptyset$
   f. $a$
   g. $\{b\}$

Recall that a topological space is sequentially compact if every sequence has a convergent subsequence.

2. a. Is the topological space in Problem 1 compact?
   b. Is it sequentially compact?
   c. Show that $K = \{0\} \cup \{1/n \mid n \in \mathbb{N}\}$ is a compact subset of $\mathbb{R}$.

   Solution: a. Yes because it is finite so every open cover is finite since there only finitely many open sets.
   b. Yes because it is finite so every sequence has a subsequence that is constant, and therefore convergent.
   c. Let $\mathcal{C}$ be an open cover. The is a $U \in \mathcal{C}$ such that $0 \in U$. We can also fine an $r > 0$ such that $B_r(0) \subseteq U$. $B_r(0)$ contains all but finitely many points of $K$ by the archimedean property. These finitely many points can be covered by a finite subcollection $\mathcal{D}$ of $\mathcal{C}$. So $\mathcal{D} \cup \{U\}$ is a finite subcover of $\mathcal{C}$.