1. Determine whether the series \( \sum_{n=1}^{\infty} \frac{2^n n!}{n^n} \) converges.

**Solution:** We apply the ratio test.

\[
\frac{2^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \frac{2(n+1)n^n}{(n+1)^{n+1}} = \frac{2}{(1+\frac{1}{n})^n} \to \frac{2}{e} < 1
\]

so the series converges.

2. Determine whether the series \( \sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n} \) converges. Hint: compare it to a \( p \)-series.

**Solution:** We have

\[
0 < \frac{\sqrt{n+1} - \sqrt{n}}{n} = \frac{(n+1) - n}{n \sqrt{n+1} + \sqrt{n}} \leq \frac{1}{n \sqrt{n+1}} = \frac{1}{2n^{3/2}}
\]

so the series converges by the comparison test since \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \) is a convergent \( p \)-series.

3. Assume the series \( \sum_{n=1}^{\infty} a_n \) converges absolutely.

a. Show that if \( p \geq 0 \) then \( \sum_{n=1}^{\infty} \frac{a_n}{n^p} \) converges absolutely.

b. Is the statement true if \( p < 0? \)

**Solution:**

a. Since \( 0 \leq \frac{|a_n|}{n^p} \leq |a_n| \) for all \( n \) the statement is true by the comparison test.

b. The statement is no longer true. For example if \( a_n = \frac{1}{n^2} \) and \( p = -1 \) then \( \sum_{n=1}^{\infty} a_n \) converges absolutely since it is a \( p \)-series with \( p = 2 > 1 \). But

\[
\sum_{n=1}^{\infty} \frac{a_n}{n^{-1}} = \sum_{n=1}^{\infty} \frac{1}{n}
\]

does not converge.