Name: **Alec Smart** Homework: ???? Date: 8/30/2015

Problem 1. Show that $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in \mathbb{R}$.

Proof. For all $a, b \in \mathbb{R}$ we have

$$\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ba + b^2 \quad \text{(since } ab = ba\text{)} \\
&= a^2 + 2ab + b^2.
\end{aligned}$$

□

Problem 2. Find two numbers with a sum of 3 and a product of 2.

Proof. Denote the two numbers by a and b. Since the sum of the numbers is 3, we must have

$$a + b = 3.$$

Since their product is 2, we must have

$$ab = 2.$$

From the first equation we have

$$a = 3 - b.$$

Substituting this into the second equation gives

$$(3 - b)b = 2.$$

This implies

$$(b-1)(b-2) = b^2 - 3b + 2 = 0.$$

Hence $b \in \{1, 2\}$. If $b = 1$ then $a = 2$. If $b = 2$ then $a = 1$. So the only possibility is that one of the numbers is 1 while the other is 2. These two numbers in fact have a sum of 3 and a product of 2. □