```
(* We find the maximum of f(x,y,z) =
 2x+y+z on the intersection of the plain x-y+2z =
  1 and the paraboloid z = x^2+2y^2 *)
f = 2 x + y + z
g = x - y + 2 z
h = x^2 + 2 y^2 - z
ContourPlot3D[f, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}]
ContourPlot3D[{g == 1, h == 0,}, {x, -2, 1.5}, {y, -1, 1}, {z, 0, 2}]
```

$2x + y + z$

$x - y + 2z$

$x^2 + 2y^2 - z$

```
Gf = Grad[f, {x, y, z}]
Gg = Grad[g, {x, y, z}]
Gh = Grad[h, {x, y, z}]
```

$\{2, 1, 1\}$

$\{1, -1, 2\}$

$\{2x, 4y, -1\}$

```
sol = Solve[{Gf == alpha Gg + beta Gh, g == 1, h == 0}]
```

$$\left\{\left\{z \to \frac{1}{48}\left(33 - \sqrt{57}\right),\ \text{alpha} \to \frac{1}{38}\left(19 + 3\sqrt{57}\right),\ \text{beta} \to 3\sqrt{\frac{3}{19}},\ x \to \frac{1}{12}\left(-3 + \sqrt{57}\right),\ y \to \frac{1}{24}\left(3 + \sqrt{57}\right)\right\},\ \left\{z \to \frac{1}{48}\left(33 + \sqrt{57}\right),\ \text{alpha} \to \frac{1}{38}\left(19 - 3\sqrt{57}\right),\ \text{beta} \to -3\sqrt{\frac{3}{19}},\ x \to \frac{1}{12}\left(-3 - \sqrt{57}\right),\ y \to \frac{1}{24}\left(3 - \sqrt{57}\right)\right\}\right\}$$

```
s1 = {x, y, z, f} /. sol[[1]]
s1 // N
```

$$\left\{\frac{1}{12}\left(-3 + \sqrt{57}\right),\ \frac{1}{24}\left(3 + \sqrt{57}\right),\ \frac{1}{48}\left(33 - \sqrt{57}\right),\ \frac{1}{48}\left(33 - \sqrt{57}\right) + \frac{1}{6}\left(-3 + \sqrt{57}\right) + \frac{1}{24}\left(3 + \sqrt{57}\right)\right\}$$

$\{0.379153, 0.439576, 0.530212, 1.72809\}$

```
s2 = {x, y, z, f} /. sol[[2]]
s2 // N
```

$$\left\{\frac{1}{12}\left(-3-\sqrt{57}\right),\ \frac{1}{24}\left(3-\sqrt{57}\right),\ \frac{1}{48}\left(33+\sqrt{57}\right),\ \frac{1}{6}\left(-3-\sqrt{57}\right)+\frac{1}{24}\left(3-\sqrt{57}\right)+\frac{1}{48}\left(33+\sqrt{57}\right)\right\}$$

{-0.879153, -0.189576, 0.844788, -1.10309}

```
(* Approximate global extrema* )
(* Global maximum of 1.728 at (0.379,0.439,0.530 *)
(* Global minimum of -1.103 at (-0.879,-0.189,0.844) *)
```